INTRODUCTION TO CIVIL ENGINEERING

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Syllabus

Analysis of force systems:

Concept of idealization, system of forces, principles of super position and transmissibility, Resolution and composition of forces, Law of Parallelogram of forces, Resultant of concurrent coplanar force systems (equilibrium of concurrent). Resultant of non-concurrent coplanar force systems, Varignon's theorem, moment of forces, couple, free body diagram, and non-concurrent coplanar force systems (beams) Numerical examples

FUNDAMENTAL LAWS IN MECHANICS

- Newton's I law

  Every body continues in its state of rest or of uniform motion in a straight line, unless it is compelled to do so by force acting on it.

- Newton's II law

  The rate of change of momentum is directly proportional to the applied force and takes place in the direction of the impressed force.

FUNDAMENTAL LAWS IN MECHANICS

- Newton's III law

  For every action there is an equal and opposite reaction.

FORCE

An action or agent, which changes or tends to change the state of rest or of uniform motion of a body.

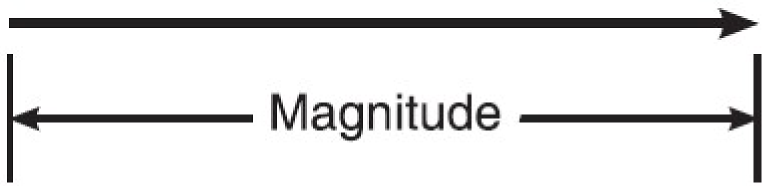
Characteristics of a force

These are ones, which help in understanding a force completely, representing a force and also distinguishing one force from one another.

CHARACTERISTICS OF A FORCE

A force is a vector quantity. It has four important characteristics, which can be listed as follows.

1) Magnitude: The length of the vector represents the magnitude of force, as shown in Figure



CHARACTERISTICS OF A FORCE

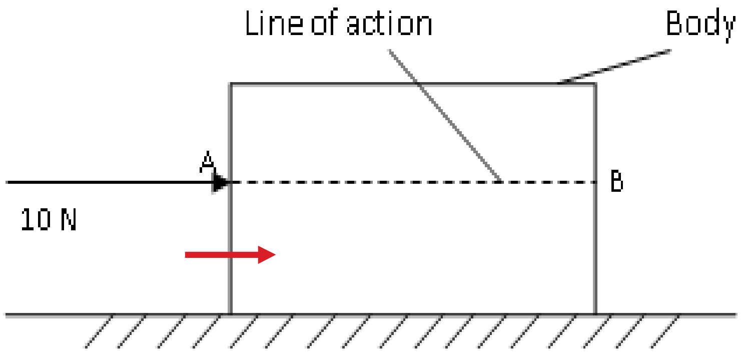
2) Point of application - It is the point at which the force acts.

3) Line of action- It is the line along which the force acts.

4) Direction- The direction of a force can be represented by an arrowhead.

COMPREHENSIVE QUESTIONS

The characteristics of the force acting on the body are



1) Magnitude is 10 N.
2) Point of application is A.
3) Line of action is A to B or AB.
4) Direction is horizontally to right.

COMPREHENSIVE QUESTIONS

Consider a ladder AB resting on a floor and leaning against a wall, on which a person weighing 750 N stands on the ladder at a point C on the ladder.

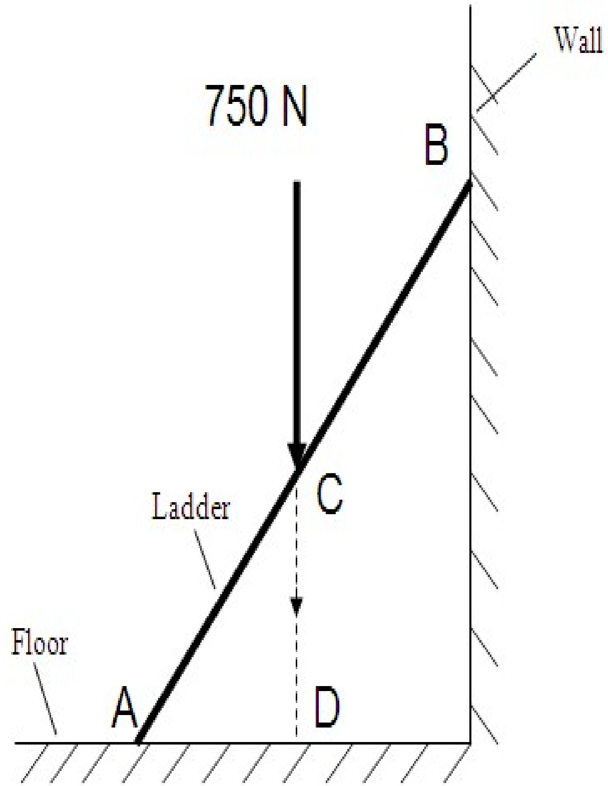
The characteristics of the force acting on the ladder are

1) Magnitude is 750 N.

2) Point of application is C.

3) Line of action is C to D or CD.

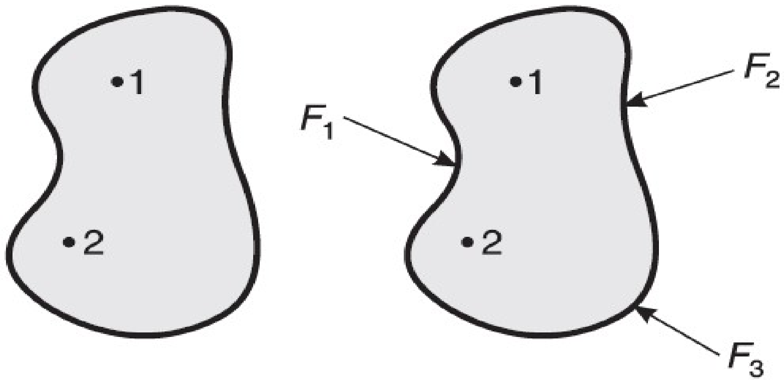
4) Direction is vertically downward.



IDEALIZATION OR ASSUMPTIONS IN MECHANICS

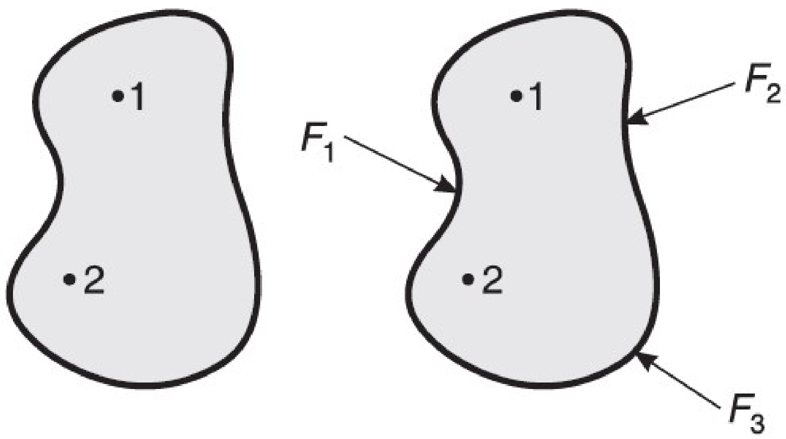
In applying the principles of mechanics to practical problems, a number of ideal conditions are assumed. They are as follows.

1) Continuum: A body consists of continuous distribution of matter.



IDEALIZATION OR ASSUMPTIONS IN MECHANICS

2) Rigid body: It is one in which the positions of the constituent particles do not change under the application of external forces, such as the position of particles 1 and 2 in Figure



IDEALIZATION OR ASSUMPTIONS IN MECHANICS

3) Particle:

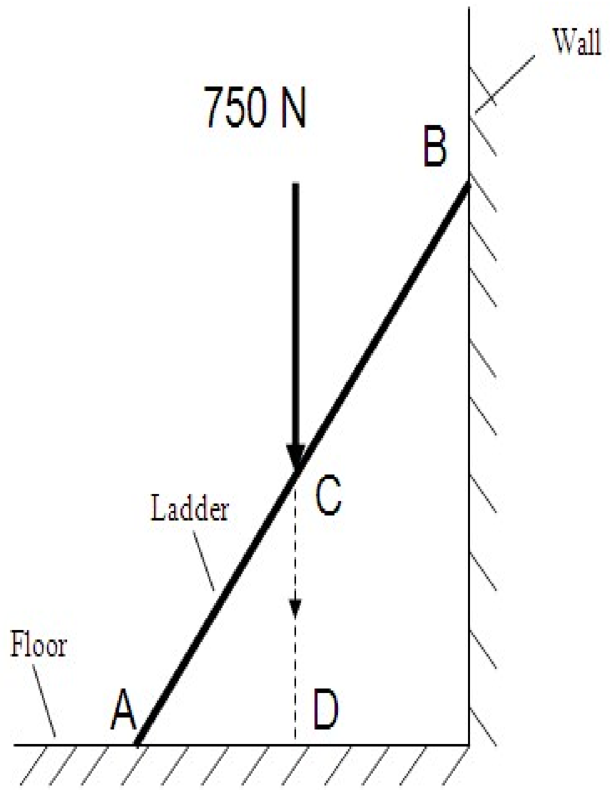
- It has mass but not size.
- A body of infinitely small volume whose mass can be neglected, is called a particle.

4) Point force:

- It is a force which is acting at a fixed point.
- Let us consider a man climbing a ladder.
- The weight of the man is not actually concentrated at a fixed point but for the purpose of analysis it is assumed to be concentrated at a particular point.

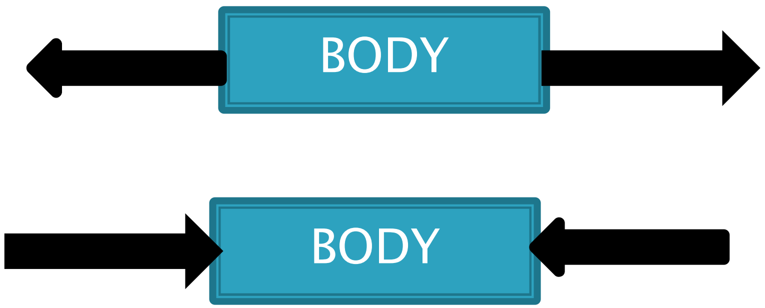
EXTERNAL FORCE

- The forces that act on a structure from the outside.
- Gravity is a non contact force that acts downward on an object
- Applied/contact forces act from the outside. These include wind, earthquakes, weight of people on the floors, weight of the building itself.



INTERNAL FORCE

- Forces that act between two different parts of a same structure are called Internal Forces.
- Four types:
- Tension (force pulling the particles of an object apart)
- Compression (squeezes the particles of an object together)



INTERNAL FORCE

- Torsion(internal twisting force created in an object as a result of a twisting motion being applied to the object)

INTERNAL FORCE

Source: 1) Jocelyn Martinez's STEM Blog
2) St. Mary's Spring Company

INTERNAL FORCE

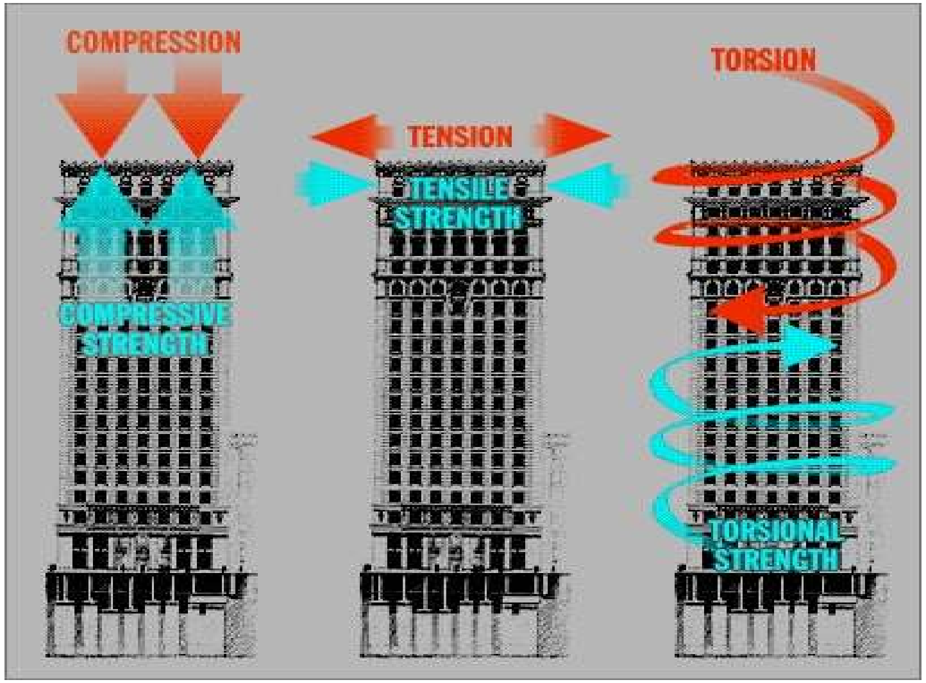
- Shear (forces acting in an object as a result of pushes and/or pulls in opposite directions; usually in rips or tears in an object)

Source: quora

Source: Andrea Rada's STEM Blog

INTERNAL FORCE

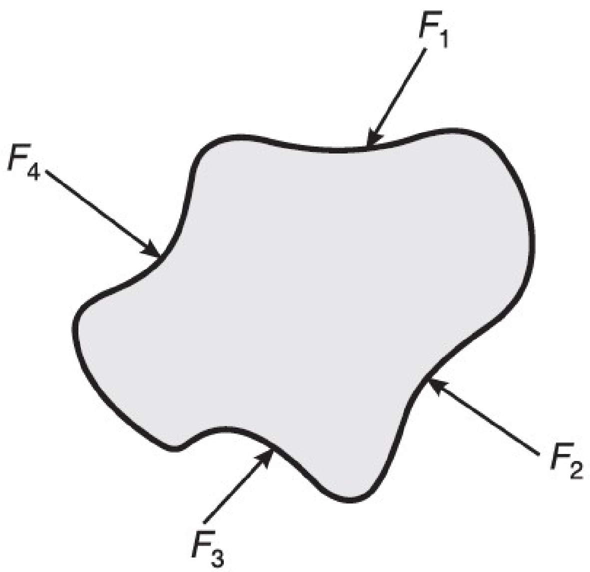
➢ Internal forces act between different parts of the same structure.



Source: http://player.slideplayer.com/23/6668221/#

FORCE SYSTEM

- If two or more forces are acting on a body or a particle, then it is said to be a force system, such as that shown in Figure



CLASSIFICATION OF FORCE SYSTEMS

The types of force system are:

1. Coplanar force system

2. Non-coplanar force system

3. Collinear and non-collinear force system

COPLANAR FORCE SYSTEM

If two or more forces are acting in a single plane, then it is said to be a coplanar force system. The types of coplanar force system are:

(i) Coplanar concurrent force system

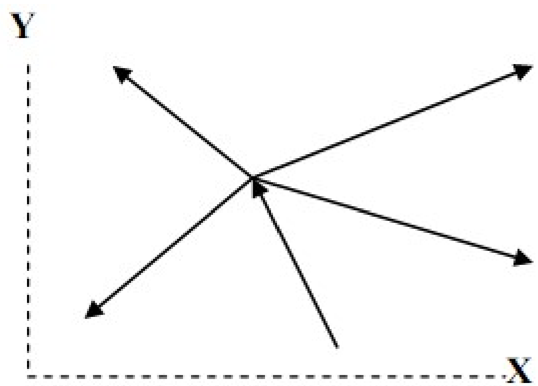
(ii) Coplanar non-concurrent force system

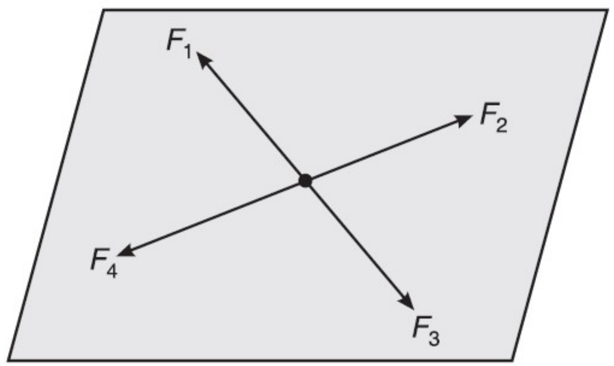
(iii) Coplanar parallel force system

COPLANAR FORCE SYSTEM

(i) Coplanar concurrent force system:

It is a force system, in which all the forces are lying in the same plane and lines of action meet a single point.



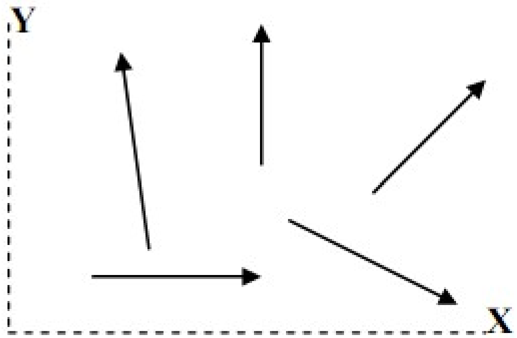


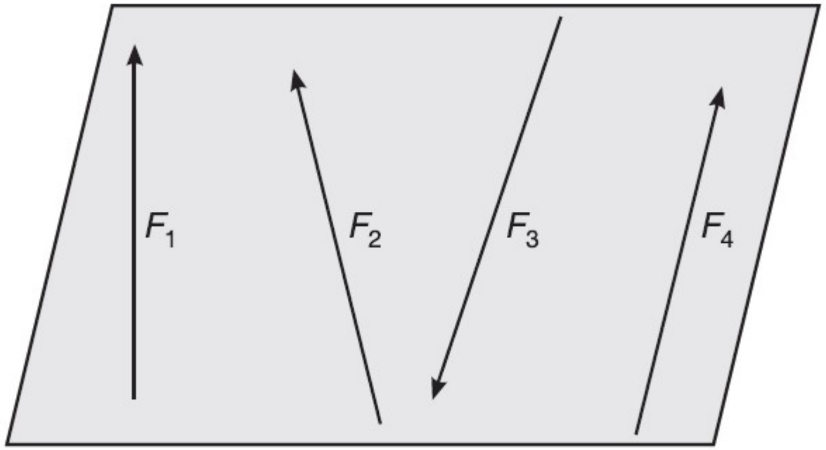
Ex.: The forces in the rope and pulley arrangement.

COPLANAR FORCE SYSTEM

(ii) Coplanar non-concurrent force system:

Coplanar non-concurrent forces: It is a force system, in which all the forces are lying in the same plane but lines of action do not meet a single point.





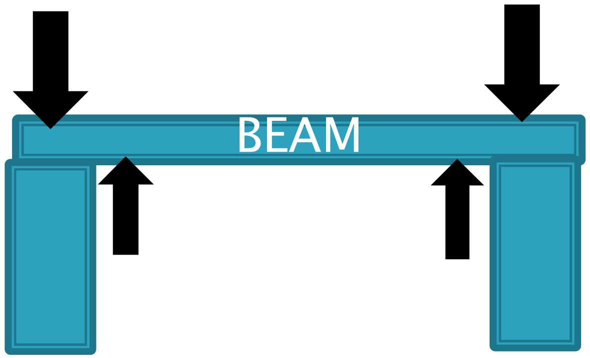
Ex.: Forces on a ladder and reactions from floor and wall, when a ladder rests on a floor and leans against a wall.

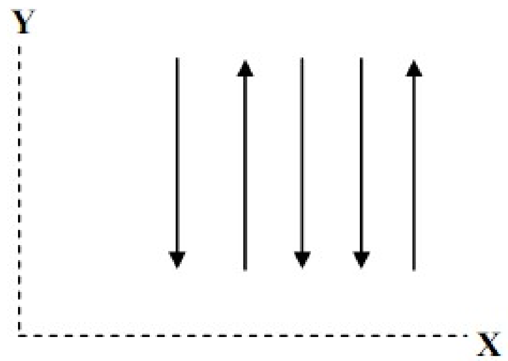
COPLANAR FORCE SYSTEM

(iii) Coplanar parallel force system:

It is a force system, in which all the forces are lying in the same plane and have parallel lines of action.

Example: System of forces acting on a beam including the reaction on the beam



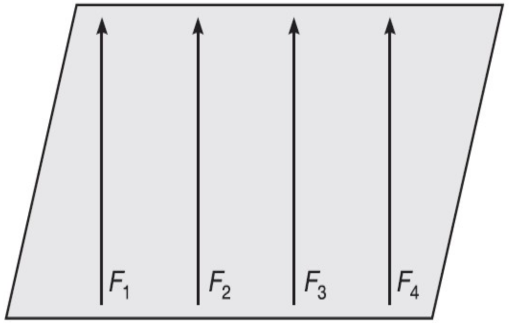


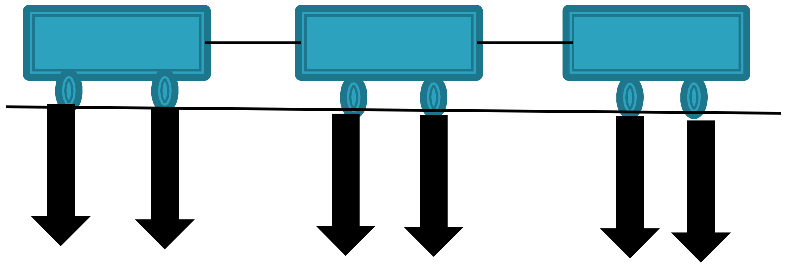
Ex.: The forces or loads and the support reactions in case of beams.

COPLANAR FORCE SYSTEM

The coplanar parallel force system is of two types:

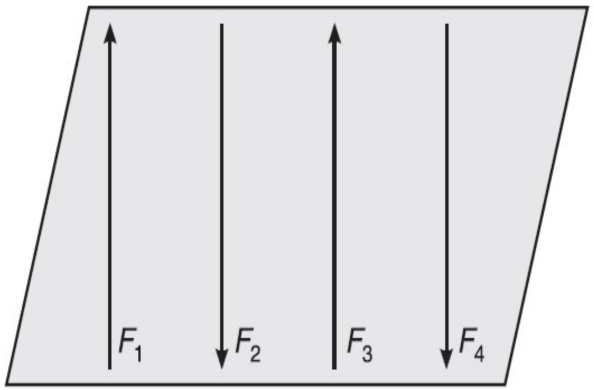
(i) Like parallel force system: All the forces act parallel to one another and are in the same direction, as shown in Figure. **Example:** Weight of a stationary train on a straight stretch of rail





COPLANAR FORCE SYSTEM

(ii) Unlike parallel force system: The forces act parallel to another, but some of the forces have their line of action in opposite directions, as shown in Figure. **Example:** System of forces acting on a beam with reactions developed



NON - COPLANAR FORCE SYSTEM

If two or more forces are acting in different planes, the forces constitute a non-coplanar force system. Such a system of forces can be

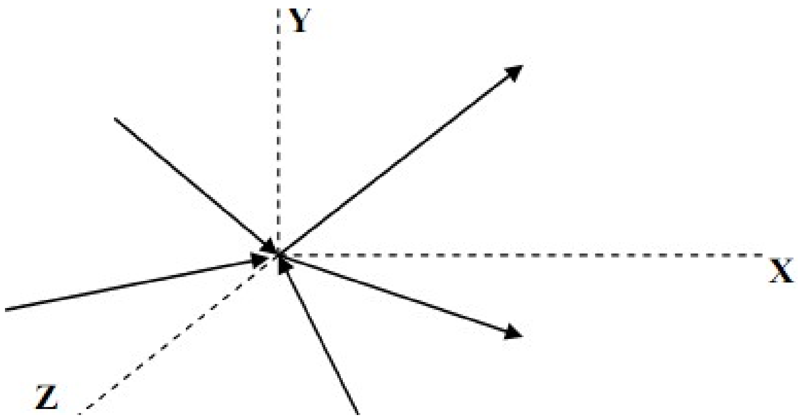
(i) Non-coplanar concurrent force system

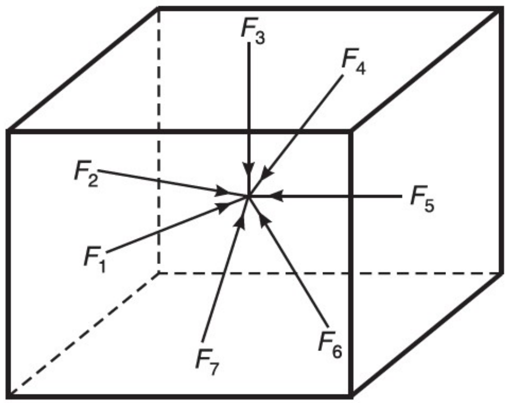
(ii) Non-coplanar non-concurrent force system

(iii) Non-coplanar parallel force system

NON - COPLANAR FORCE SYSTEM

(i) **Non-coplanar concurrent force system:** It is a force system, in which all the forces are lying in the different planes and still have common point of action. **Example:** Tripod carrying a camera





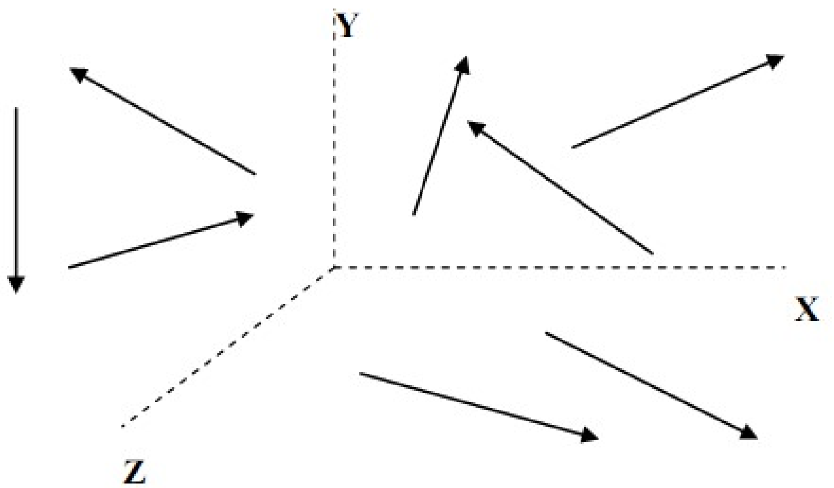
Ex.: The forces acting on a tripod when a camera is mounted on a tripod.

NON - COPLANAR FORCE SYSTEM

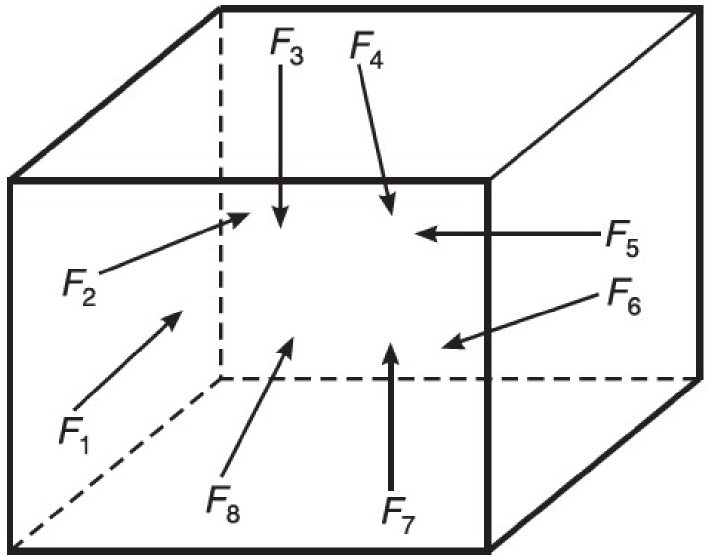
Non-coplanar concurrent force system:

NON - COPLANAR FORCE SYSTEM

(ii) Non-coplanar non-concurrent force system: It is a force system, in which all the forces are lying in the different planes and also do not meet a single point. **Example:** Forces acting on a moving bus

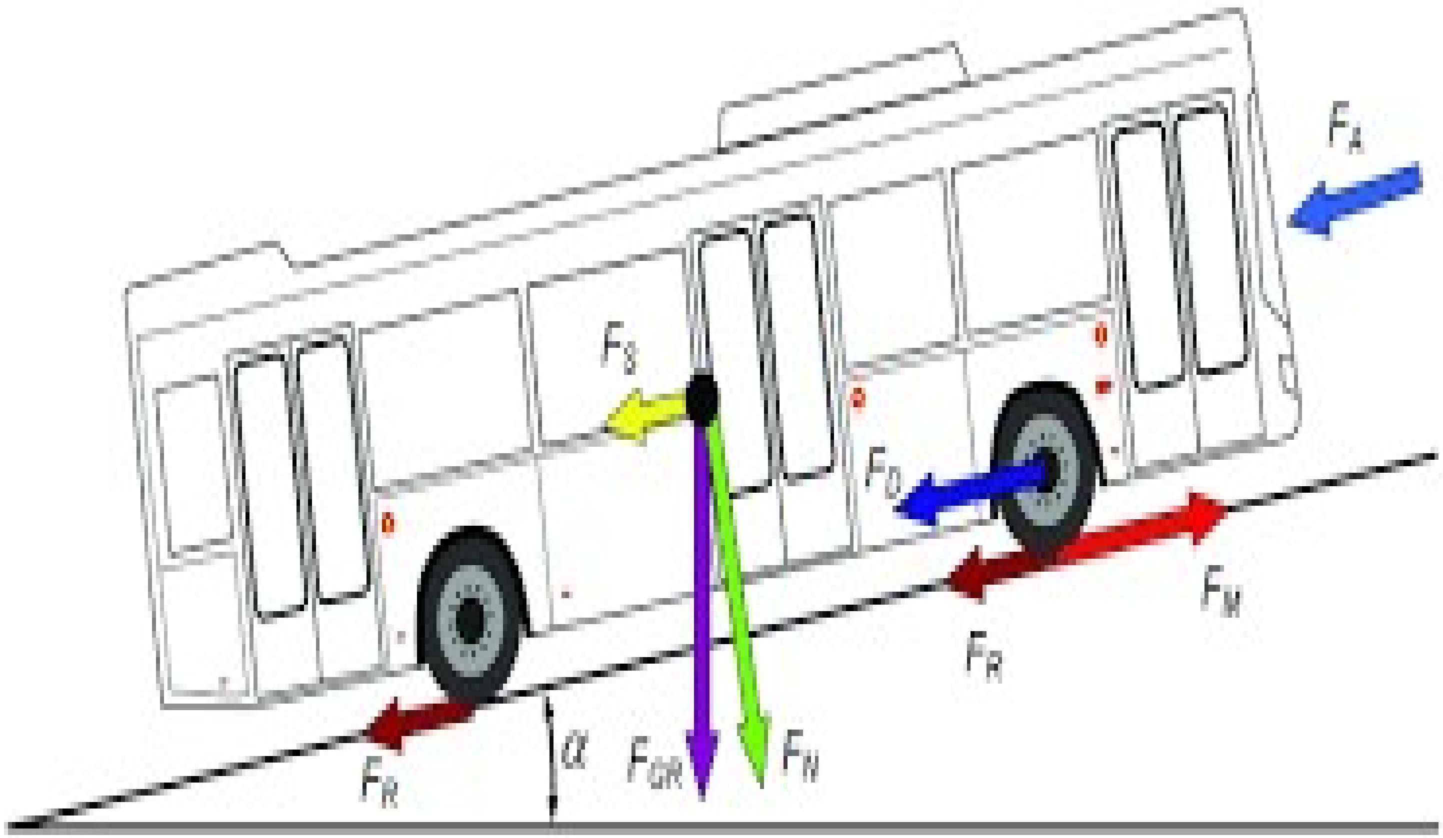


Ex.: Forces acting on a building fr



NON - COPLANAR FORCE SYSTEM

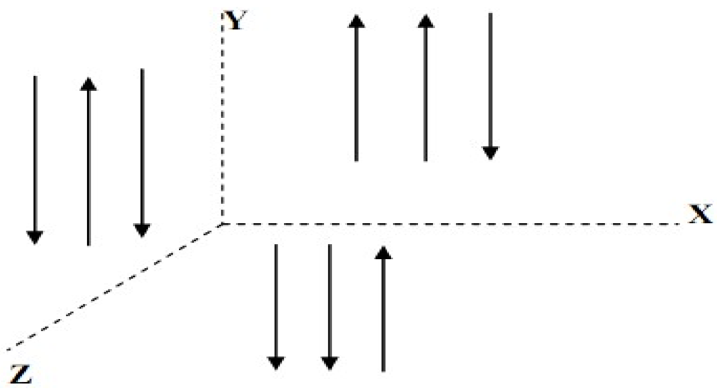
(ii) Non-coplanar non-concurrent force system:

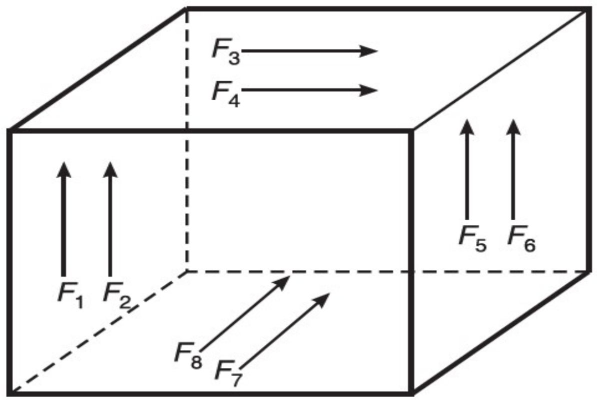


Source: ResearchGate

NON - COPLANAR FORCE SYSTEM

(iii) Non-coplanar parallel force system: It is a force system, in which all the forces are lying in the different planes and still have parallel lines of action. Example: Weight of benches in a class room





Ex: The forces acting and the reactions at the points of contact of bench with floor in a classroom.

NON - COPLANAR FORCE SYSTEM

(iii) Non-coplanar parallel force system:

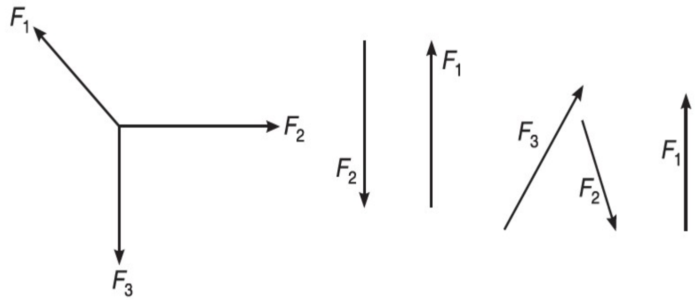
Source: Pinterest

COLLINEAR AND NON-COLLINEAR FORCE SYSTEM

(i) Collinear force system: If the lines of action of two or more forces coincide with one another, it is called a collinear force system.

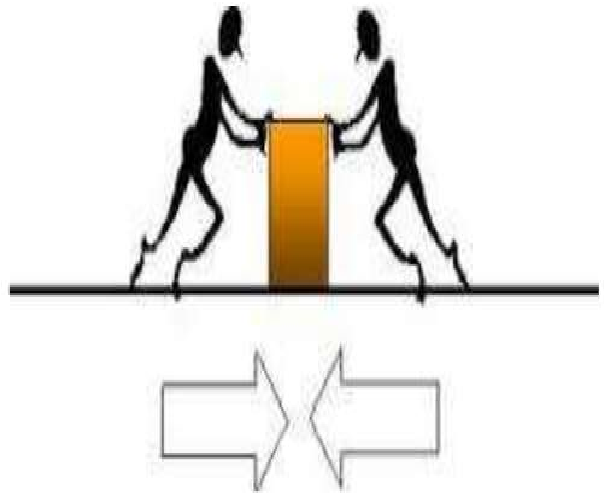
(ii) Non-collinear force system: If the lines of action of the forces do not coincide with one another, it is called a non-collinear force system.

Ex.: Forces in a rope in a tug of war



PRINCIPLE OF SUPERPOSITION OF FORCES

This principle states that "the net effect of given system of forces on a rigid body is not changed by adding or subtracting another system of forces in equilibrium."



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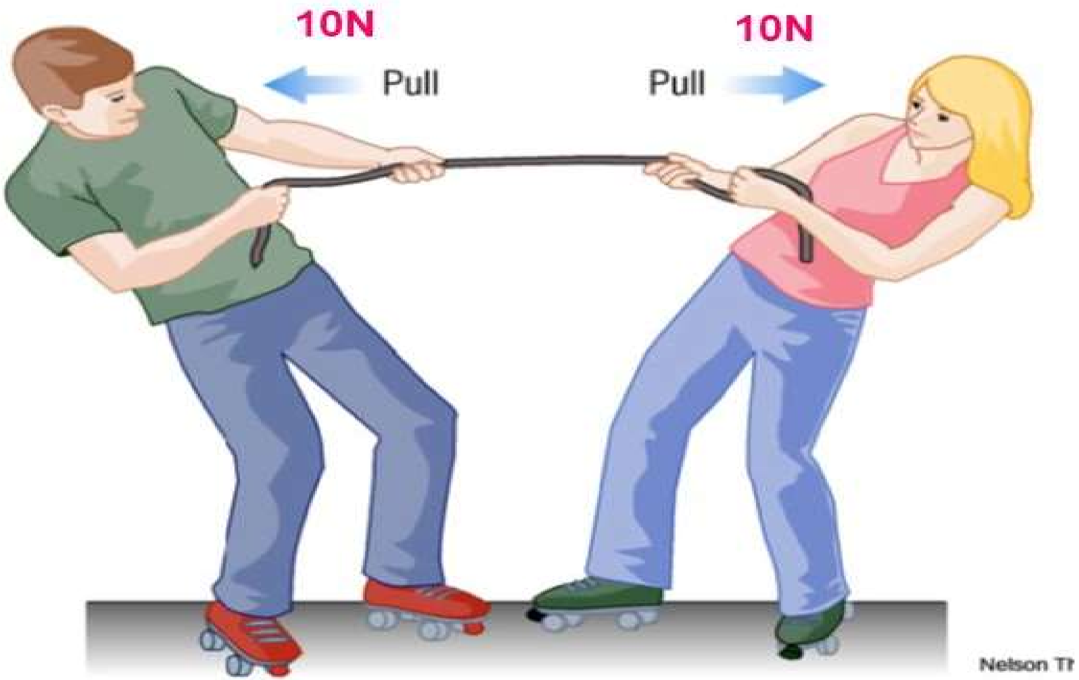
PRINCIPLE OF SUPERPOSITION OF FORCES

P2 2.1 Forces Between objects

When two objects interact (push or pull), they exert equal and opposite forces on each other.

The skaters move towards each other

What happens?



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PRINCIPLE OF SUPERPOSITION OF FORCES

EXAMPLE: Tug of war

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PRINCIPLE OF PHYSICAL INDEPENDENCE OF FORCES

This principle states that ‘the action of a force on a body is not affected by the action of any other force on the body’.

Example: Newtons third law

A boat being rowed in a lake

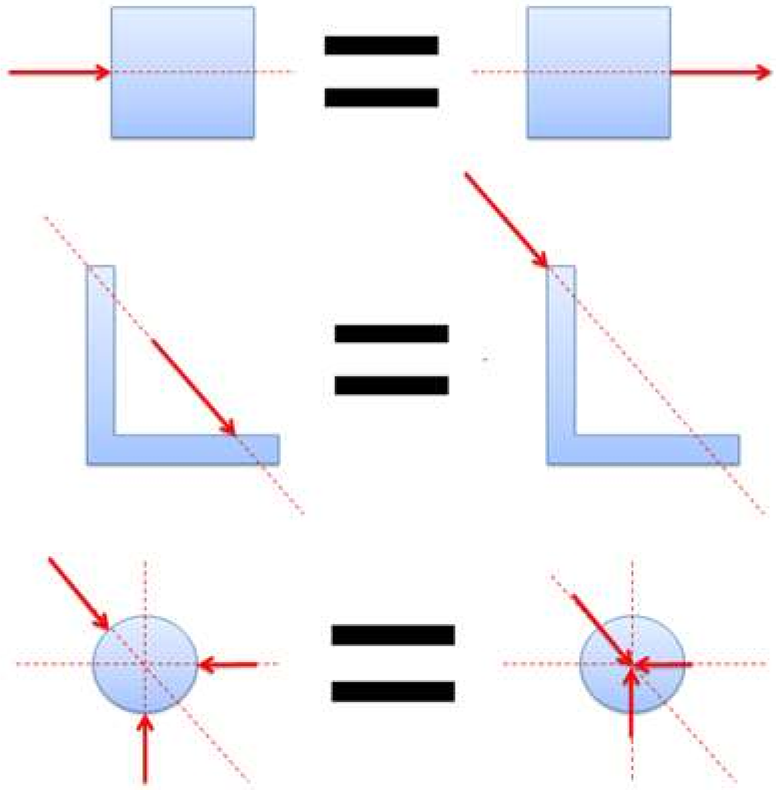
PRINCIPLE OF TRANSMISSIBILITY OF FORCES

The state of rest or of uniform motion of a rigid body is unaltered if the point of application of the force is transmitted to any other point along the line of action of the force.

Disadvantage: Not applicable for deformable bodies

PRINCIPLE OF TRANSMISSIBILITY OF FORCES

Examples:



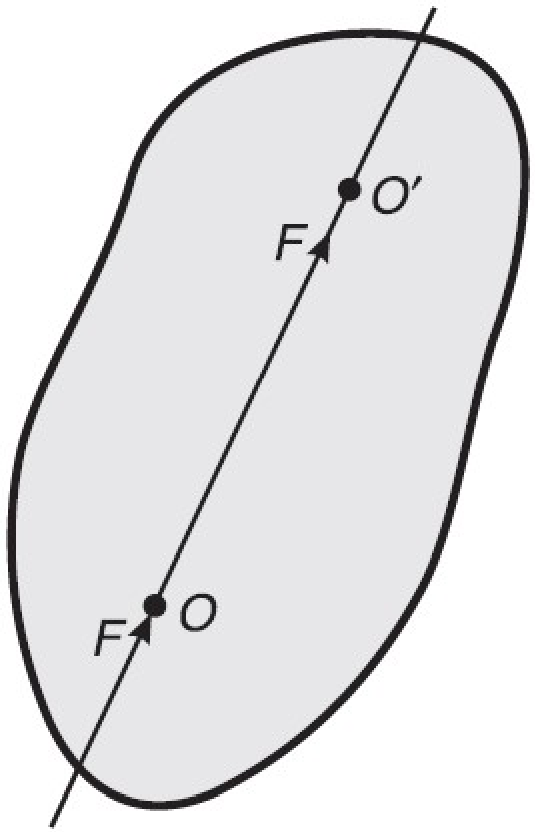
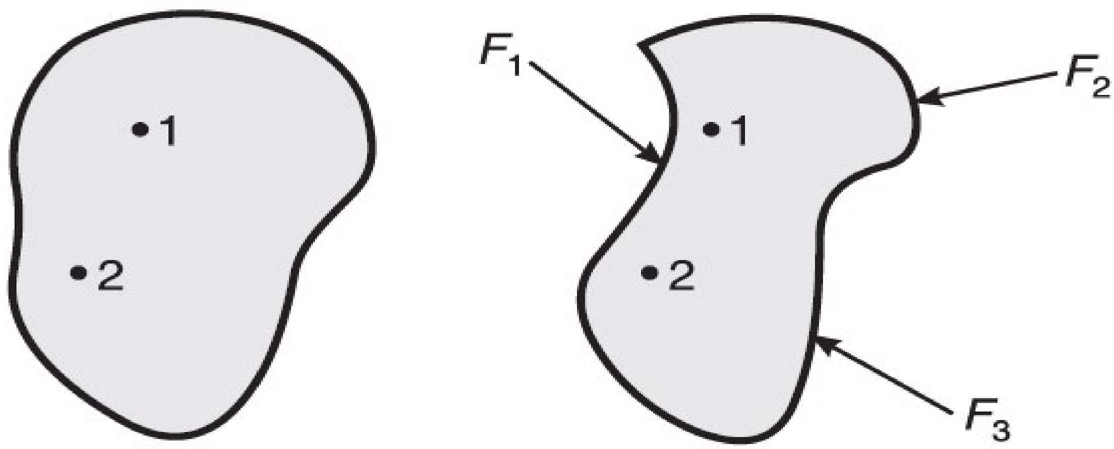


Fig. Transmissibility of force F from point O to O'.

DEFORMABLE BODY

It is the one in which the positions of constituent particles change under the application of external forces, such as the positions of particles 1 and 2 in Figure.



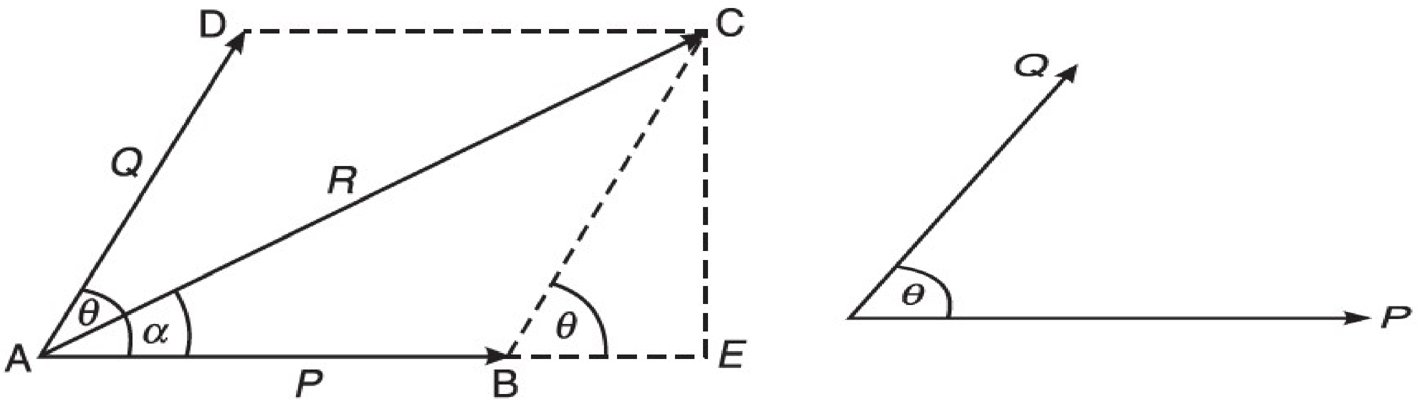
METHODS OF FINDING THE RESULTANT

The resultant of a system of coplanar concurrent forces can be determined by the following methods.

1) Parallelogram law
2) Triangle law
3) Polygon law
4) Method of Resolution

PARALLELOGRAM LAW

If two forces are acting simultaneously on a particle and away from the particle, with the two adjacent sides of the parallelogram representing both the magnitude and direction of forces, the magnitude and direction of the resultant can be represented by the diagonal of the parallelogram starting from the common point of the two forces.

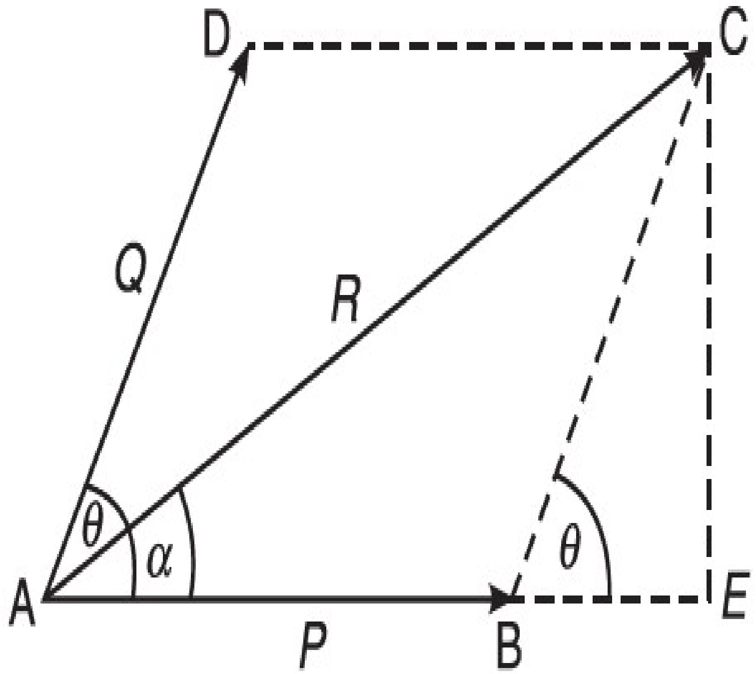


Let *P* and *Q* be the two forces, represented by the sides AB and AD of the parallelogram, the resultant can then be represented by AC as shown below:

To find the magnitude *R* of the resultant, consider the ΔCAE, where

$$AC^2 = AE^2 + CE^2$$

$$= (AB + BE)^2 + (CE$$



Consider the Δ CBE, where

$$CE = Q \sin \theta$$

$$BE = Q \cos \theta$$

$$AC^2 = AB^2 + 2AB.BE + BE^2 + CE^2$$

$$R^2 = P^2 + 2.\ P.\ Q \cos\theta + Q^2 \cos^2\theta + Q^2 \sin^2\theta$$

$$= P^2 + Q^2 + 2.\ P.\ Q \cos\theta$$

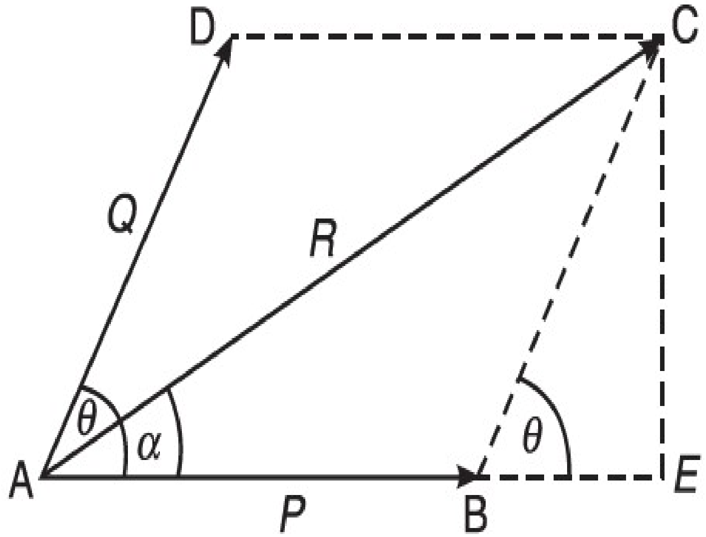
$$R = \text{Sq Rt}\ (P^2 + Q^2 + 2.\ P.\ Q \cos\theta\ )$$

To find the direction α of the resultant, consider the Δ CAE,

where

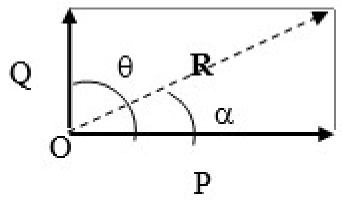
$$\tan\alpha = \frac{CE}{AB + BE}$$

$$= \frac{Q\sin\theta}{P + Q\cos\theta}$$



DIFFERENT CASES OF PARALLELOGRAM LAW

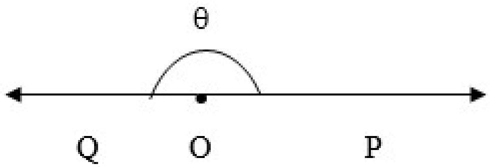
Case 1: When $\theta = 90^0$:



$$R = \sqrt{P^2 + Q^2}$$

$$\alpha = \tan^{-1}\left[\frac{Q}{P}\right]$$

Case 2: When $\theta = 180^0$:



$$R = [P - Q]$$

$$\alpha = 0^0$$

Case 3: When $\theta = 0^0$:

$$R = [P + Q]$$

$$\alpha = 0^0$$

RESOLUTION OF FORCE

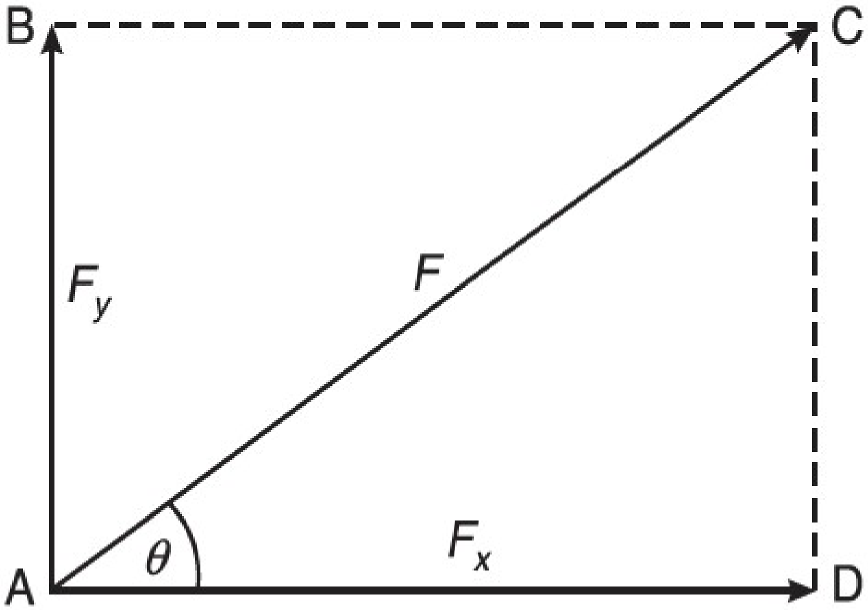
- The process of splitting up the given force into two mutually perpendicular directions is called Resolution of forces.

or

- The process of splitting of a force into its two rectangular components (horizontal and vertical) is known as resolution of the force.

RESOLUTION OF FORCE

In this figure, F is the force which makes an angle θ with the horizontal axis, and has been resolved into two components, namely F_x and F_y, along the x-axis and y-axis



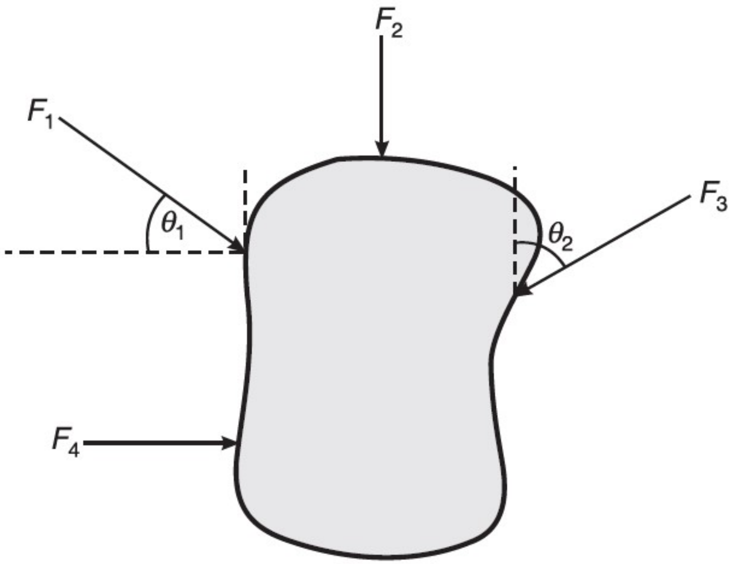
In Δ CAD,

$$\cos\theta = \frac{F_x}{F} \quad \Rightarrow \quad F_x = F\cos\theta$$

$$\sin\theta = \frac{F_y}{F} \quad \Rightarrow \quad F_y = F\sin\theta$$

COMPOSITION OF FORCE

- It is the process of combining a number of forces into a single force such that the net effect produced by the single force is equal to the algebraic sum of the effects produced by the individual forces. The single force in this case is called the **resultant force** which produces the same effect on the body as that produced by the individual forces acting together.



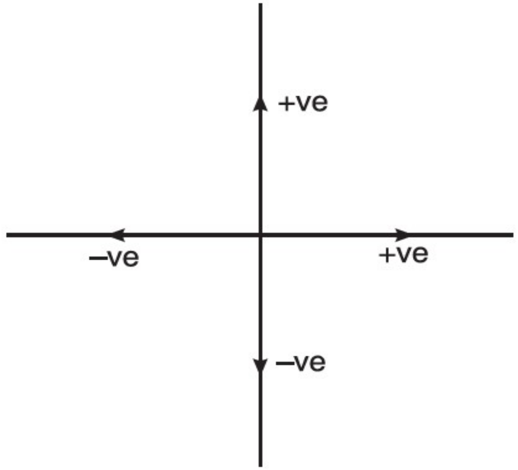


Fig. Positive and negative convention of forces

ΣF_x = algebraic sum of the components of the forces along the x-axis

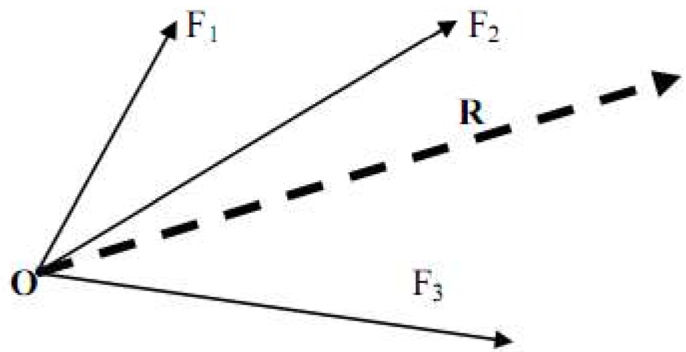
ΣF_y = algebraic sum of the components of the forces along the y-axis

The magnitude of the resultant, $R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$

and the direction of the resultant, $\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$

RESULTANT FORCE

Whenever a number of forces are acting on a body, it is possible to find a single force, which can produce the same effect as that produced by the given forces acting together. Such a single force is called as resultant force or resultant.



In the above figure R can be called as the resultant of the given forces F1, F2 and F3.

COPLANAR CONCURRENT FORCE SYSTEM

- If two or more forces are acting in a single plane and passing through a single point, such a force system is known as coplanar concurrent force system.
- In a coplanar concurrent force system, we can calculate the magnitude and direction of the resultant.

COPLANAR CONCURRENT FORCE SYSTEM

- The position, however, cannot be determined because all forces are meeting at a common point. Thus,

The magnitude of resultant, $R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$

and the direction of the resultant, $\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$

RECALLING OF EARLIER CONCEPTS

1. Calculate the algebraic sum of all the forces acting in the x- direction (i.e. ΣF_x) and also in the y-direction (i.e. ΣF_y)
2. Determine the magnitude of the resultant using the formula,

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$$

3. Determine the direction of the resultant using the formula,

$$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$$

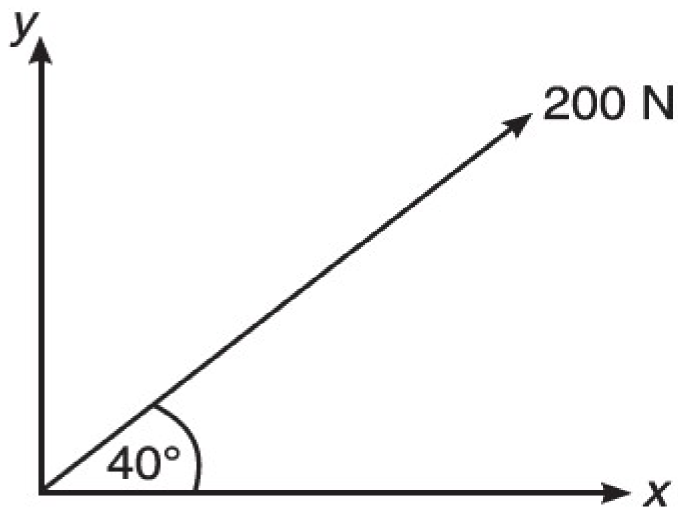
NUMERICALS

A force of 200 N is acting at a point making an angle of 40° with the horizontal as shown in Figure. Determine the components of this force along the *x* and *y* directions.

Solution

Component along the *x*-direction

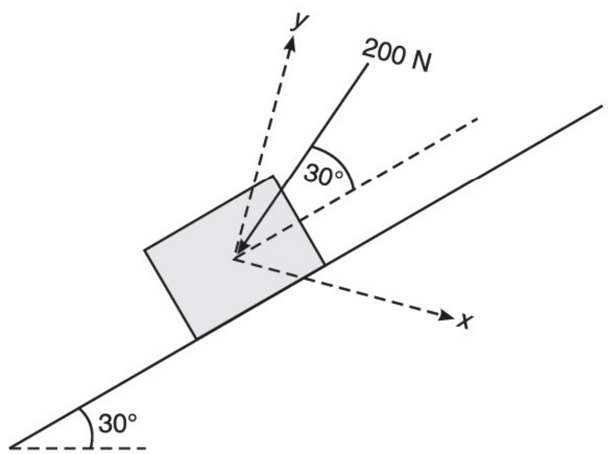
$$F_x = F \cos \theta$$
$$= 200 \times \cos 40° = 153.208 \text{ N}$$

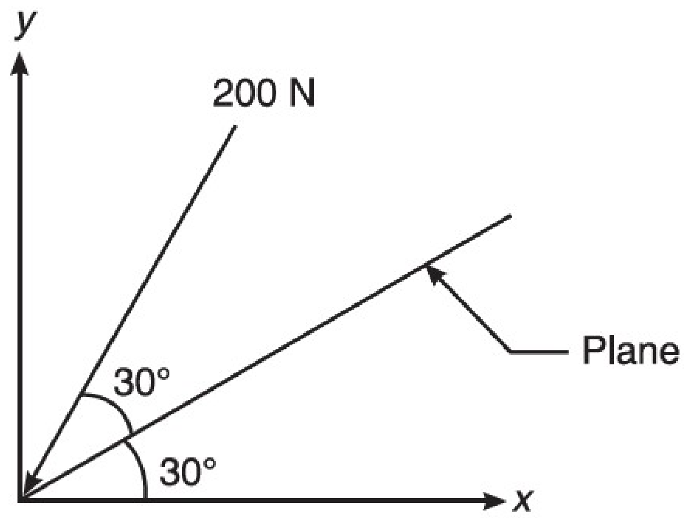


Component along the *y*-direction,

$$F_y = F \sin \theta$$
$$= 200 \times \sin 40° = 128.557 \text{ N}$$

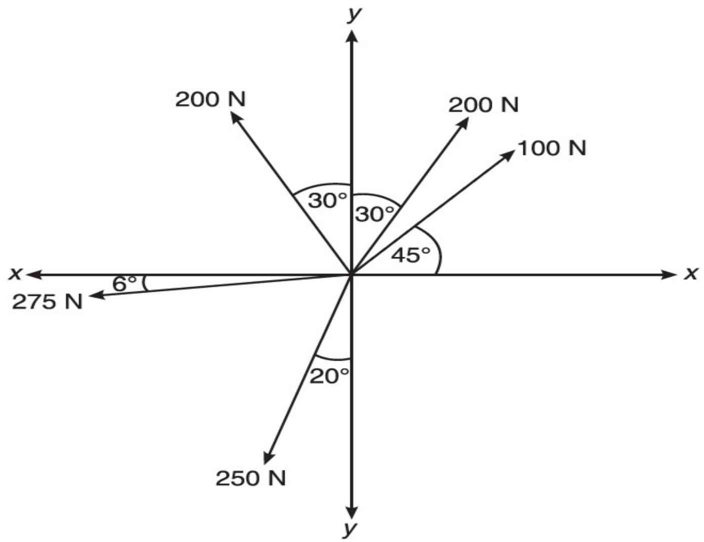
2)A force of 200 N is acting on a block as shown in Figure. Find the components of the force along the horizontal and vertical axes.





$$F_x = -200 \cos 60° = -100 \text{ N}$$
$$F_y = -200 \sin 60° = -173.2 \text{ N}$$

3) Five coplanar forces are acting at a point as shown in Figure. Determine the resultant in magnitude and direction.



$$\Sigma F_x = -200 \sin 30° - 275 \cos 6° - 250 \sin 20° + 100 \cos 45° + 200 \sin 30° = -288.287 \text{ N}$$

$$\Sigma F_y = 200 \cos 30° - 275 \sin 6° - 250 \cos 20° + 100 \sin 45° + 200 \cos 30° = 153.452 \text{ N}$$

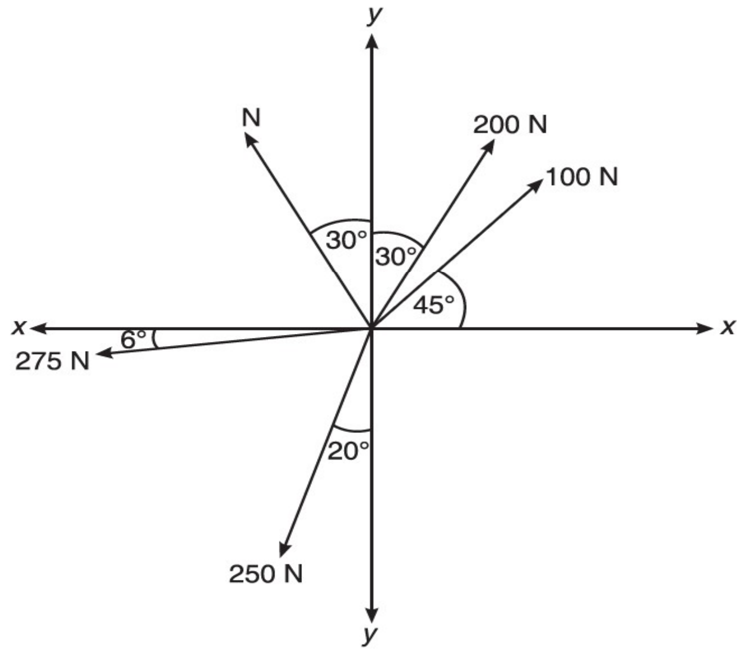
3) Solution **contd..**

$$R = \sqrt{(-288.287)^2 + (153.452)^2} = 326.584 \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$$

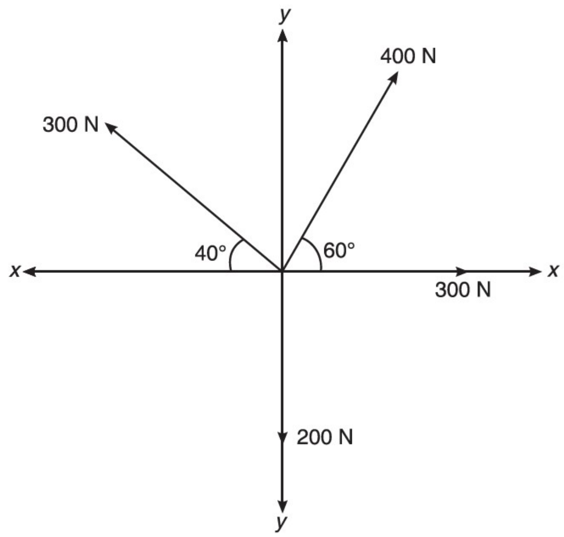
$$\theta = \tan^{-1}\left(\frac{153.452}{-288.287}\right)$$

$$\theta = 28.02°$$



4) Find the resultant of the coplanar concurrent force system shown in Figure

$$\Sigma F_x = 300 \cos 0° + 400 \cos 60° - 300 \cos 40°$$
$$= 300 + 200 - 229.8133 = 270.187 \text{ N}$$



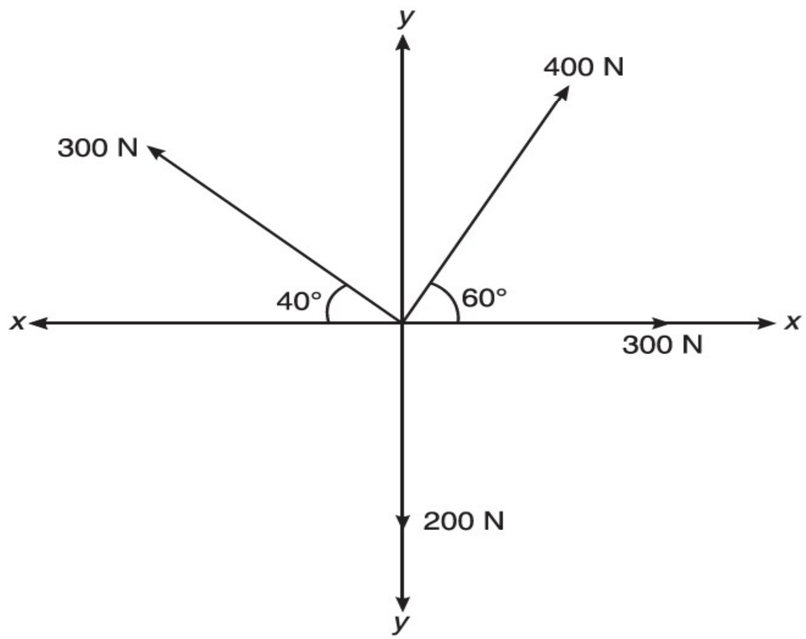
$$\Sigma F_y = 400 \sin 60° + 300 \sin 40° - 200$$
$$= 346.4102 + 192.8363 - 200 = 339.246 \text{ N}$$

4) Solution **contd..**

$$R = \sqrt{(270.187)^2 + (339.246)^2} = 433.692 \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{339.246}{270.187}\right)$$

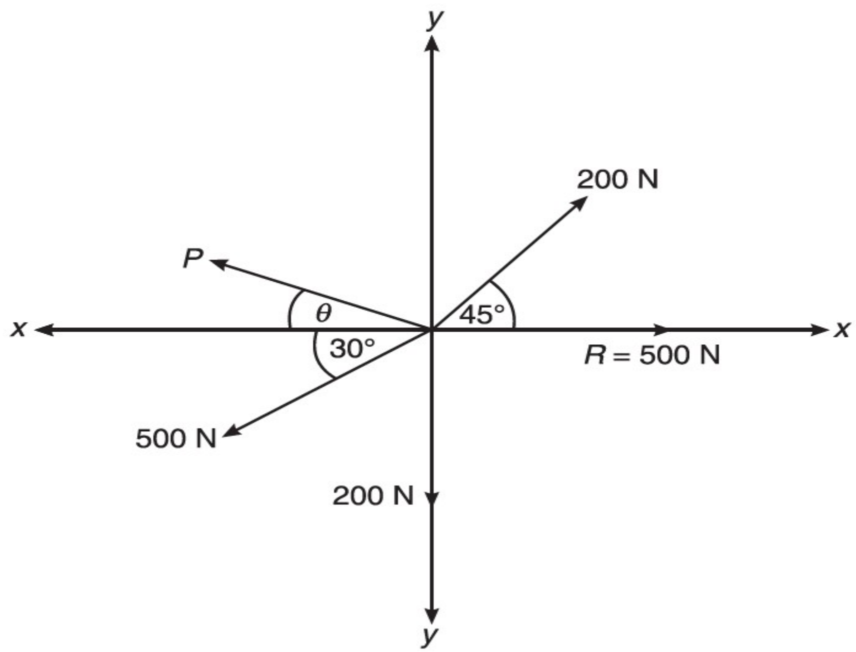
$$\theta = \tan^{-1}(1.256) = 51.47^\circ$$



5) Four coplanar forces acting at a point are shown in Figure . One of the forces is unknown and its magnitude is shown by *P*. The resultant has a magnitude of 500 N and is acting along the *x*-axis. Determine the unknown force *P* and its inclination with the *x*-axis.

We know that

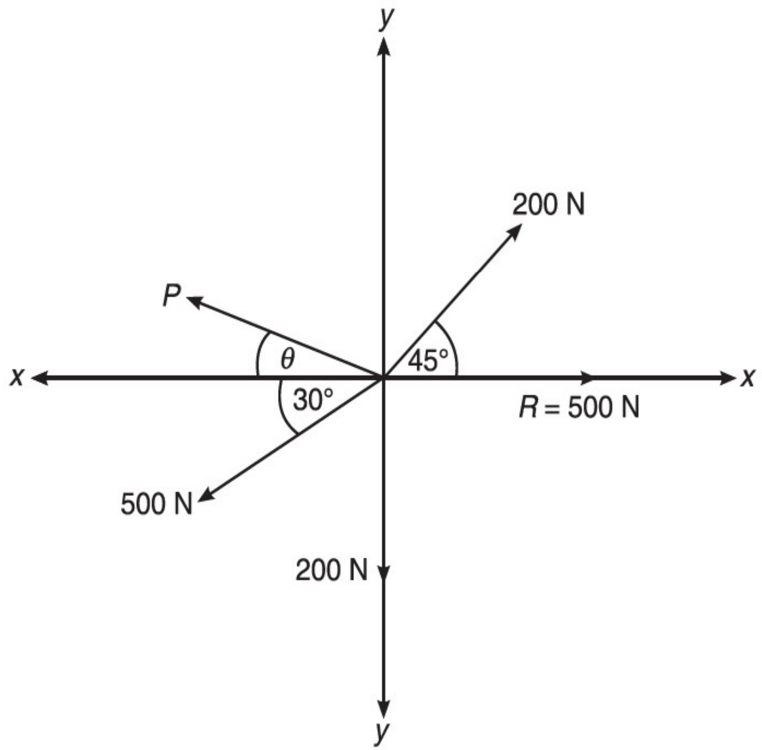
$$\Sigma F_x = R_x$$
$$\Sigma F_y = R_y$$



5) Solution **contd..**

Resolving forces along the *x*-direction,

$$\Sigma F_x = R_x = R\cos\theta = R$$
$$\Sigma F_x = 500\text{ N}$$
$$-P\cos\theta + 200\cos 45° - 500\cos 30° = 500$$
$$-P\cos\theta + 291.591 = 500$$
$$P\cos\theta = -791.59\text{ N}$$



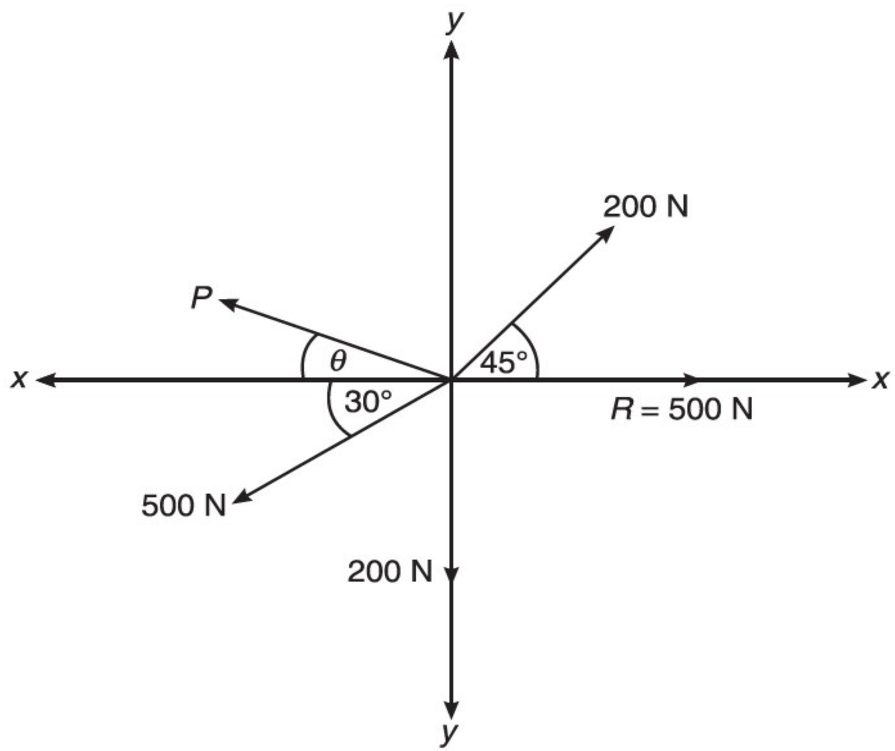
5) Solution **contd..**

$$\Sigma F_y = R_y = 0$$

$$P \sin \theta + 200 \sin 45° - 500 \sin 30° - 200 = 0$$

$$P \sin \theta - 308.579 = 0$$

$$P \sin \theta = 308.579$$



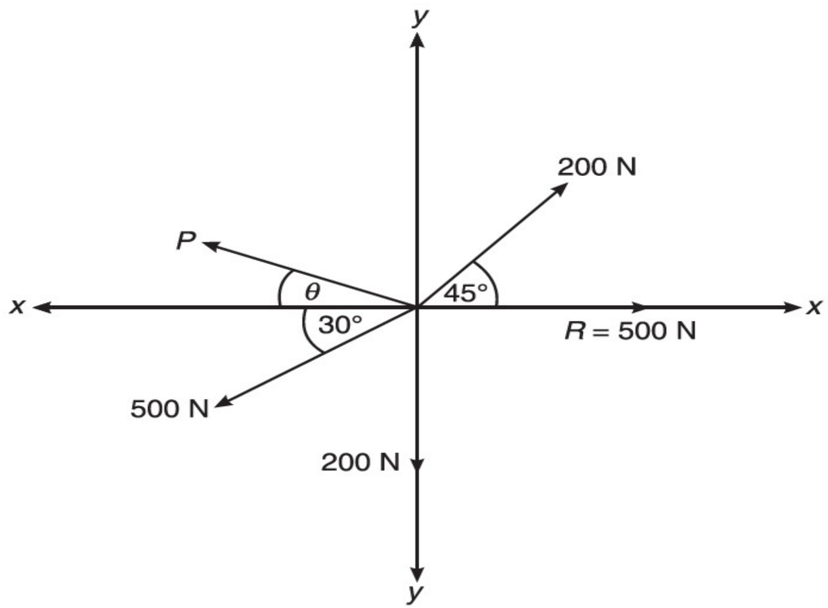
5) Solution **contd..**

Squaring both equations (i) and (ii) and then adding,

$$P^2 \cos^2\theta + P^2 \sin^2\theta = (-791.591)^2 + (308.579)^2$$

$$P^2 = 721837.31$$

$$P = 849.61 \text{ N}$$

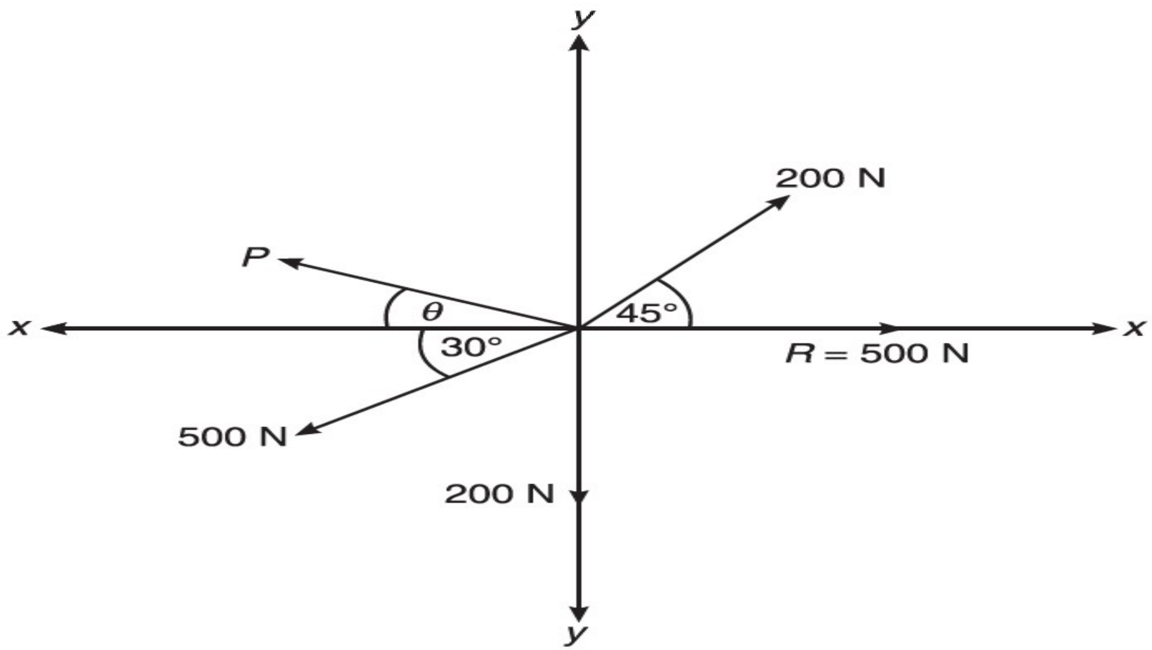


5) Solution **contd..**

Dividing Eq. (ii) by Eq. (i) gives

$$\frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{308.579}{-791.591}$$

$$\theta = \tan^{-1}\left(\frac{308.579}{-791.591}\right) = 21.297°$$



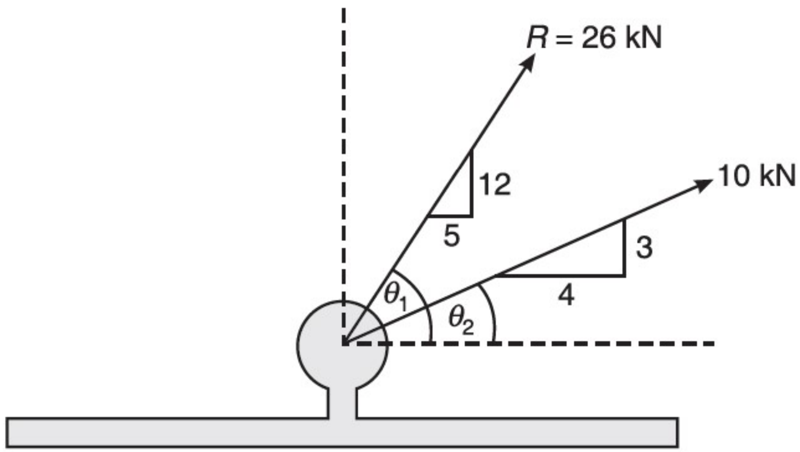
6) 26 kN force is the resultant of the two forces, one of which is as shown in Figure. Determine the other force.

Let *P* be the unknown force, which makes an angle θ with the horizontal.

R = 26 kN

$$\theta_1 = \tan^{-1}\left(\frac{12}{5}\right) = 67.38°$$

$$\theta_2 = \tan^{-1}\left(\frac{3}{4}\right) = 36.87°$$



$$\Sigma F_x = R_x$$

$$P \cos \theta + 10 \cos 36.87° = 26 \cos 67.38°$$

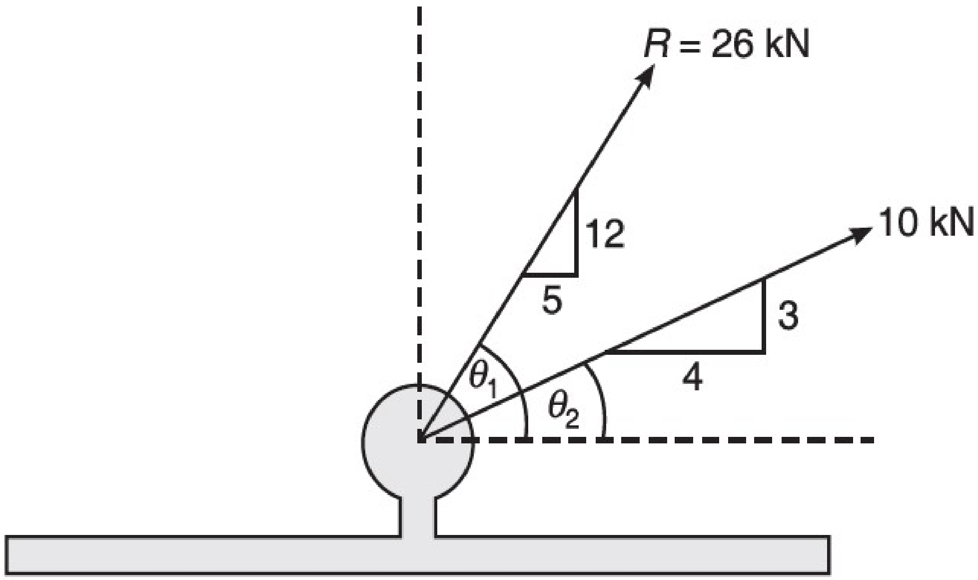
$$P \cos \theta = 26 \cos 67.38° - 10 \cos 36.87° = 2$$

6) Solution **contd..**

$$\Sigma F_y = R_y$$

$$P \sin \theta + 10 \sin 36.86° = 26 \sin 67.38°$$

$$P \sin \theta = 26 \sin 67.38° - 10 \sin 36.86° = 18$$



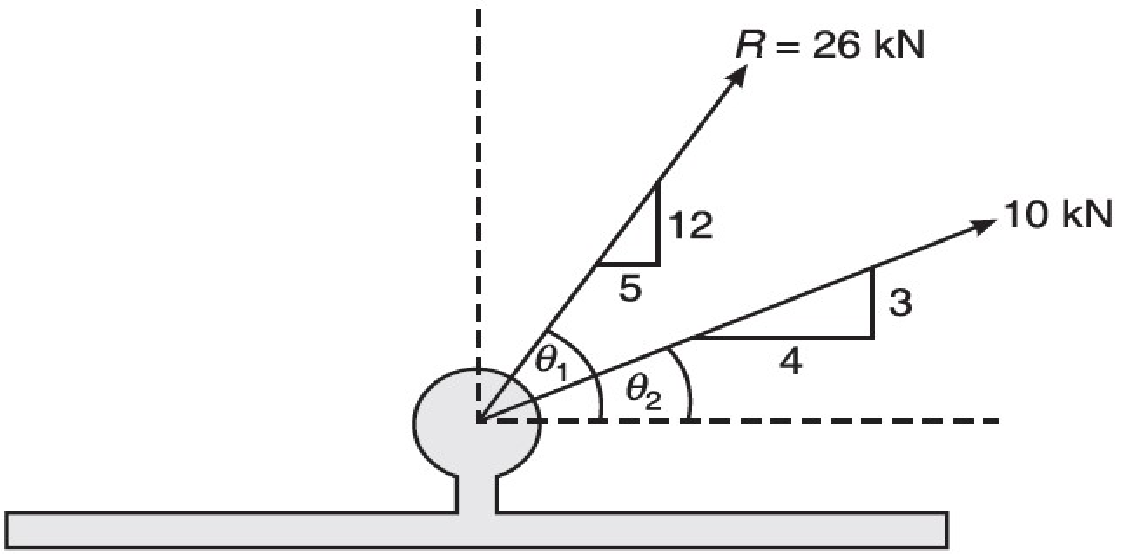
6) Solution **contd..**

Dividing Eq. (ii) by Eq. (i) gives

$$\frac{P\sin\theta}{P\cos\theta} = \frac{18}{2}$$

$$\tan\ \theta = 9$$

$$\theta = \tan^{-1}(9) = 83.66^\circ$$



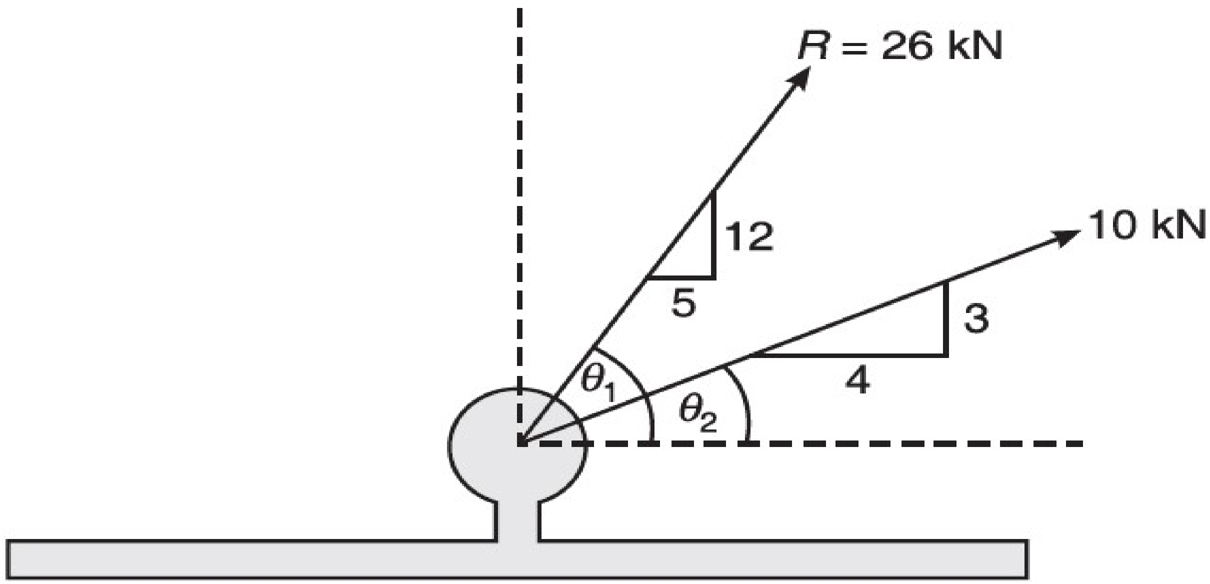
6) Solution **contd..**

Squaring (i) and (ii) and then adding

$$P^2 \sin^2\theta + P^2 \cos^2\theta = 4 + 324 = 328$$

$$P^2 = 328$$

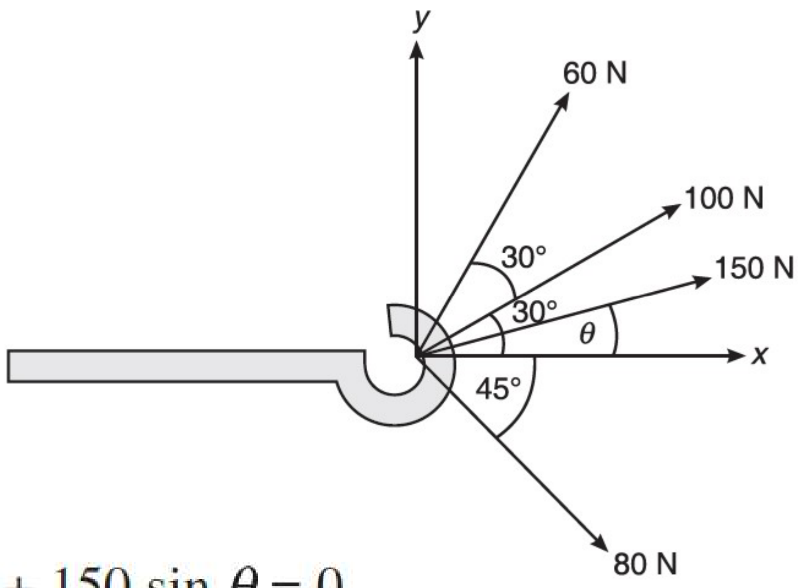
$$P = 18.11 \text{ kN}$$



7) Four forces acting on a hook are shown in Figure. Determine the direction of the force 150 N such that the hook is pulled in the *x*-direction. Determine the resultant force in the *x*-direction.

$$\Sigma F_x = R$$

$$\Sigma F_y = 0$$



For $\Sigma F_y = 0$, we have

$$-80 \sin 45° + 60 \sin 60° + 100 \sin 30° + 150 \sin \theta = 0$$

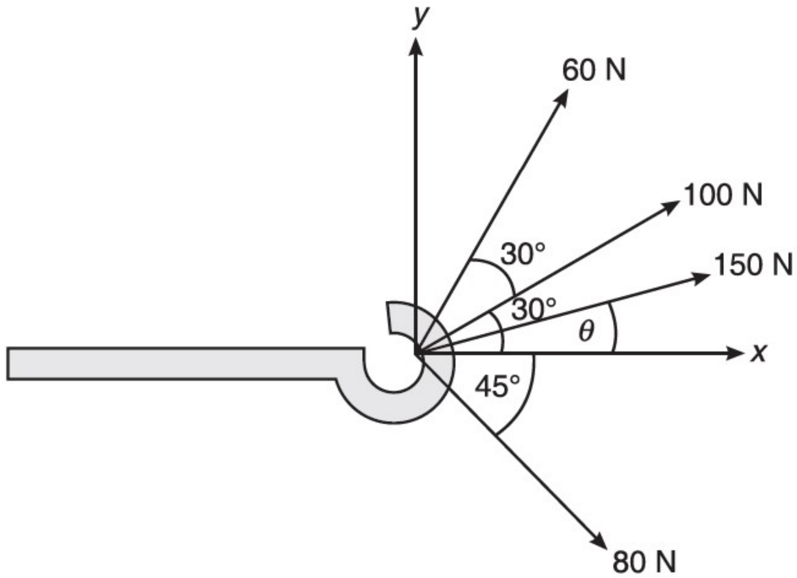
$$150 \sin \theta = 45.39$$

$$\theta = \sin^{-1}\left(\frac{45.39}{150}\right) = 17.61°$$

For $\Sigma F_x = R$, we have

$$80 \cos 45° + 60 \cos 60° + 100 \cos 30° + 150 \cos 17.61° = R$$

$$R = 316.142 \text{ N}$$



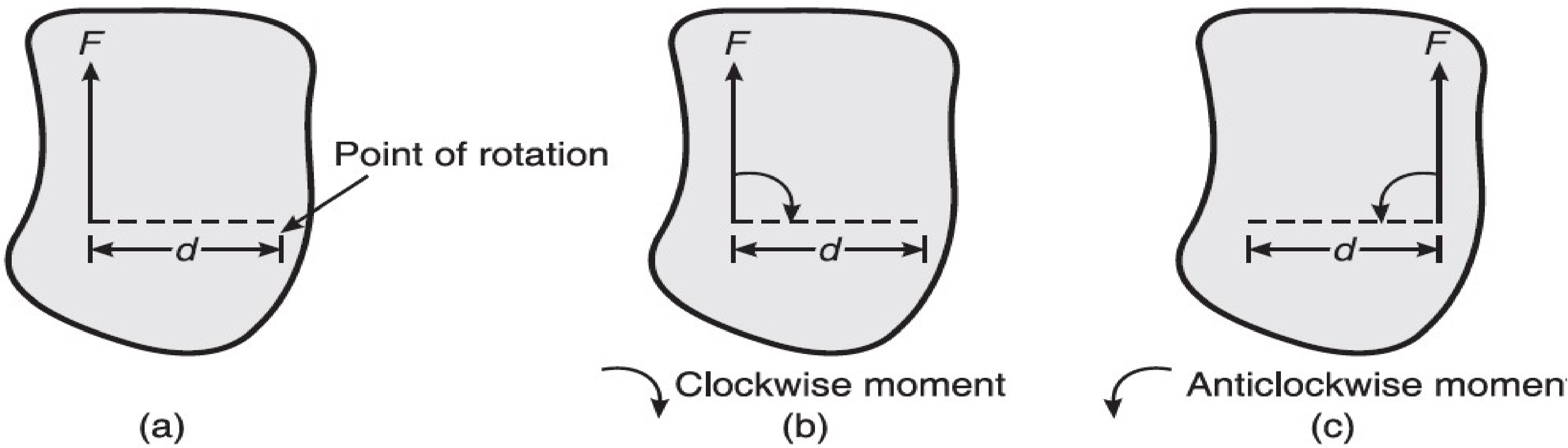
MOMENT OF A FORCE

- The turning effect produced by a force on a body is known as the moment of the force.
- The magnitude of the moment is given by the product of the magnitude of the force and the perpendicular distance between the line of action of the force and the point or axis of rotation.
- M = F x d

TYPES OF MOMENTS

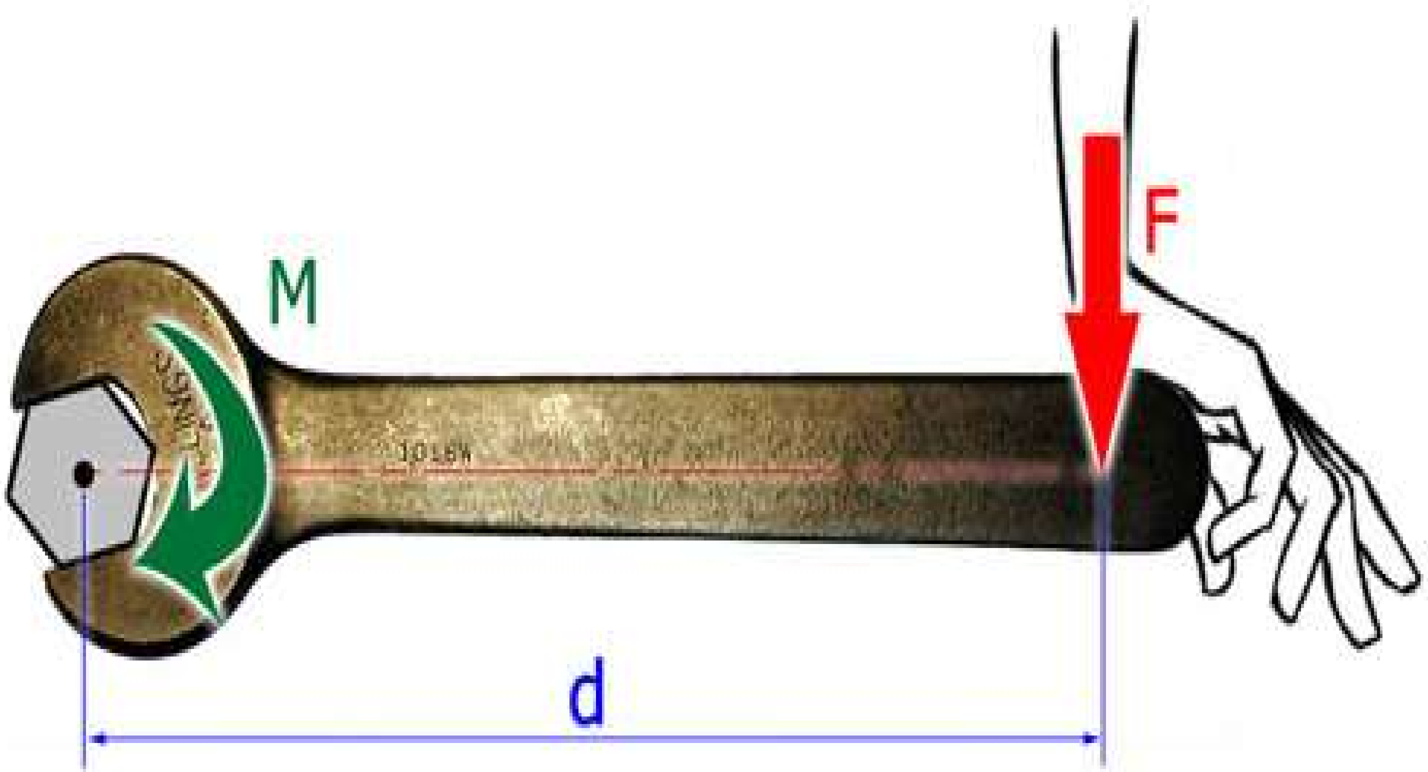
(i) If the tendency of a force is to rotate the body in the clockwise direction, it is said to be a clockwise moment and is taken positive, as shown in Figure (a) and (b)

(ii) If the tendency of a force is to rotate the body in the anticlockwise direction, it is said to be anticlockwise moment and is taken negative as shown in Figure (c)



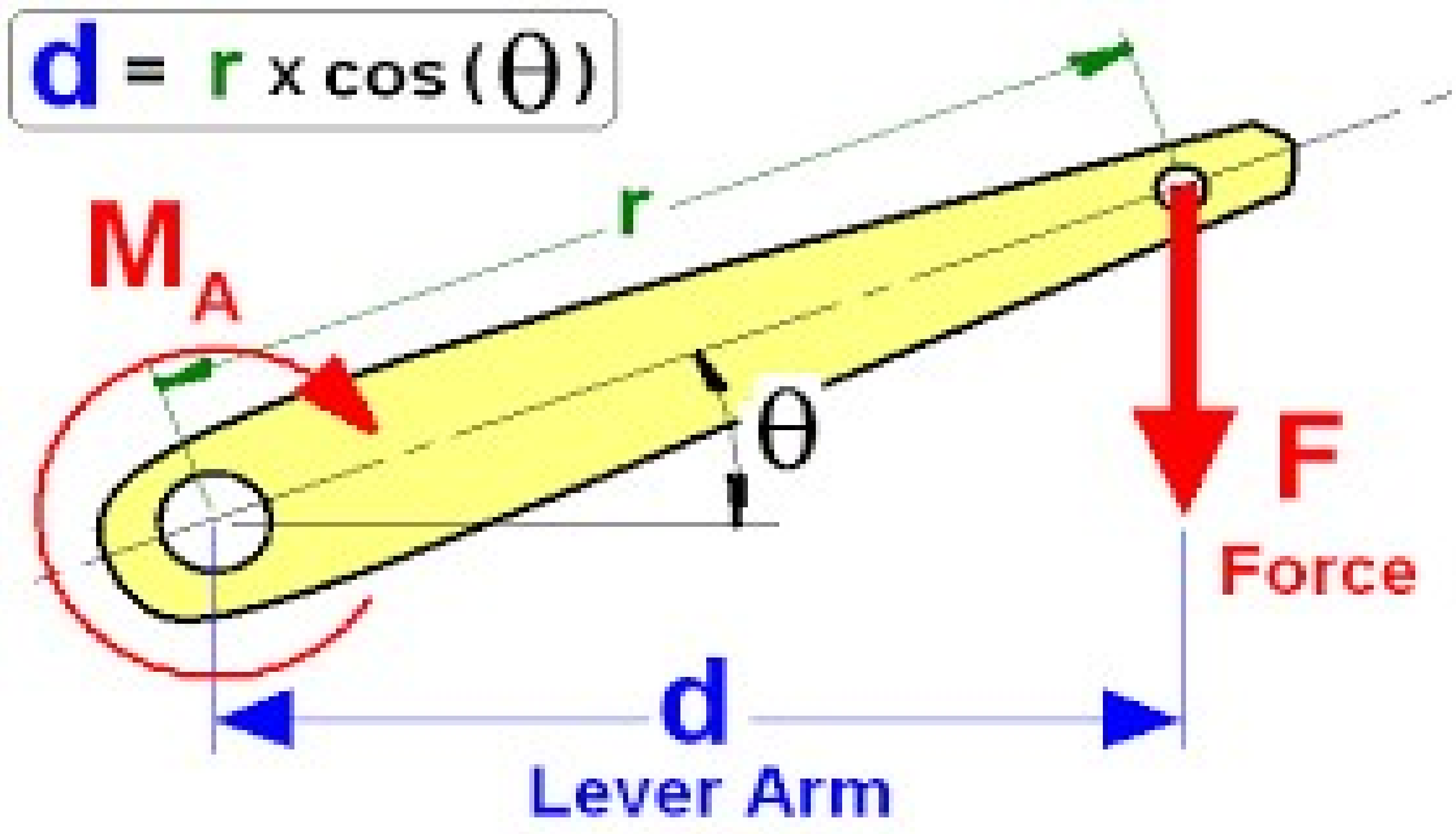
MOMENT OF A FORCE

Examples: M = F x d



Source: LearnEasy

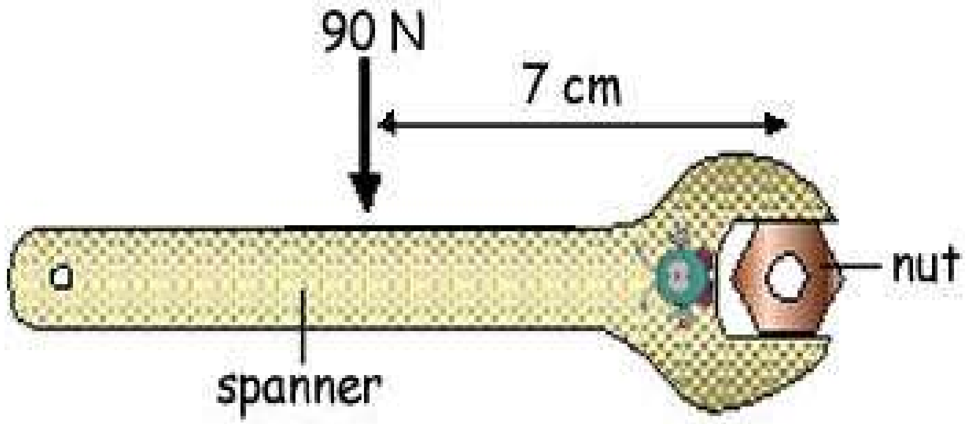
Examples:



Source: LearnEasy

COMPREHENSIVE QUESTIONS

Determine the moment of force for the given figure



Express the unit for moment of a force for the above figure

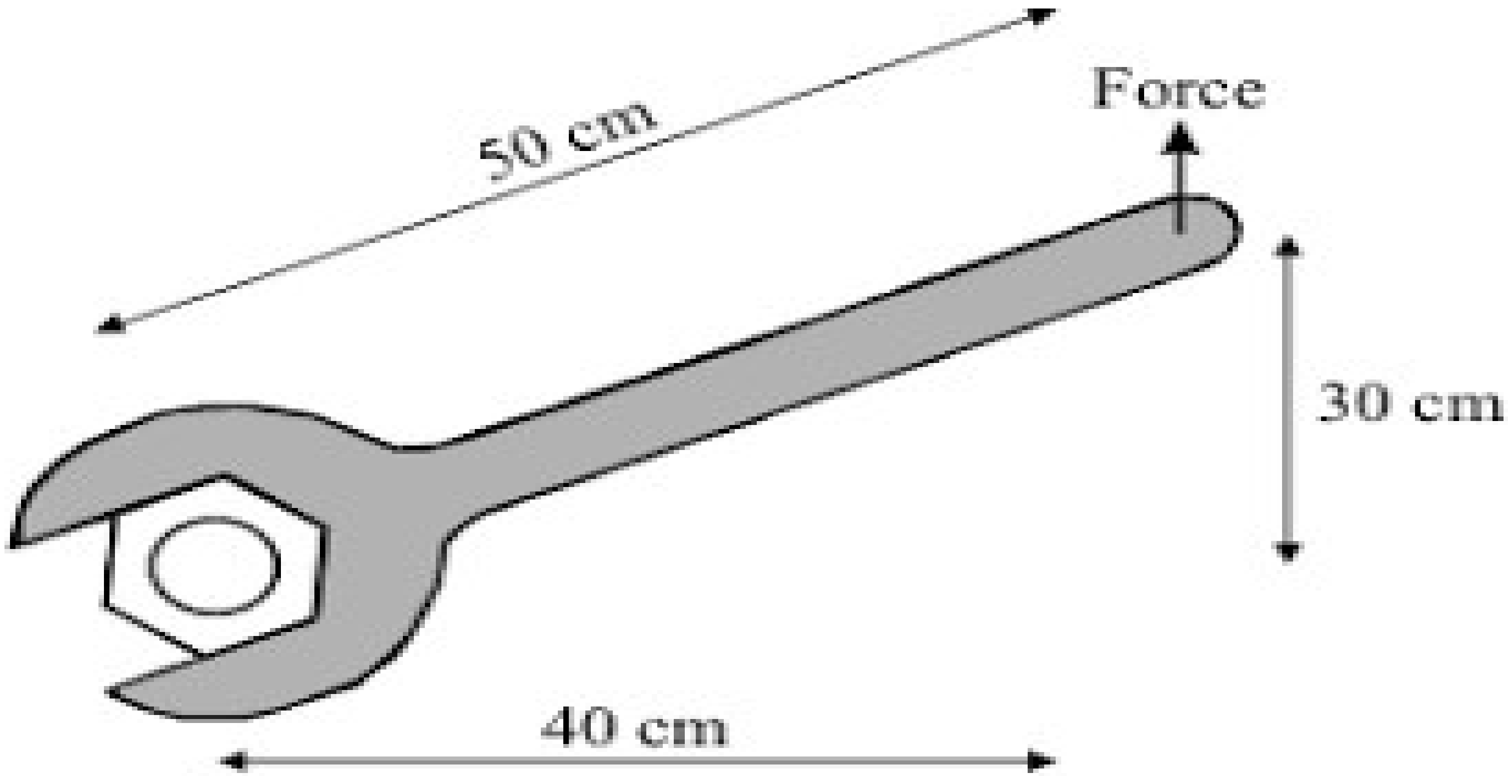
M = F x d

= N x cm

M = N - cm

COMPREHENSIVE QUESTIONS

Determine the magnitude of force for the given figure by assuming moment as 20N-cm



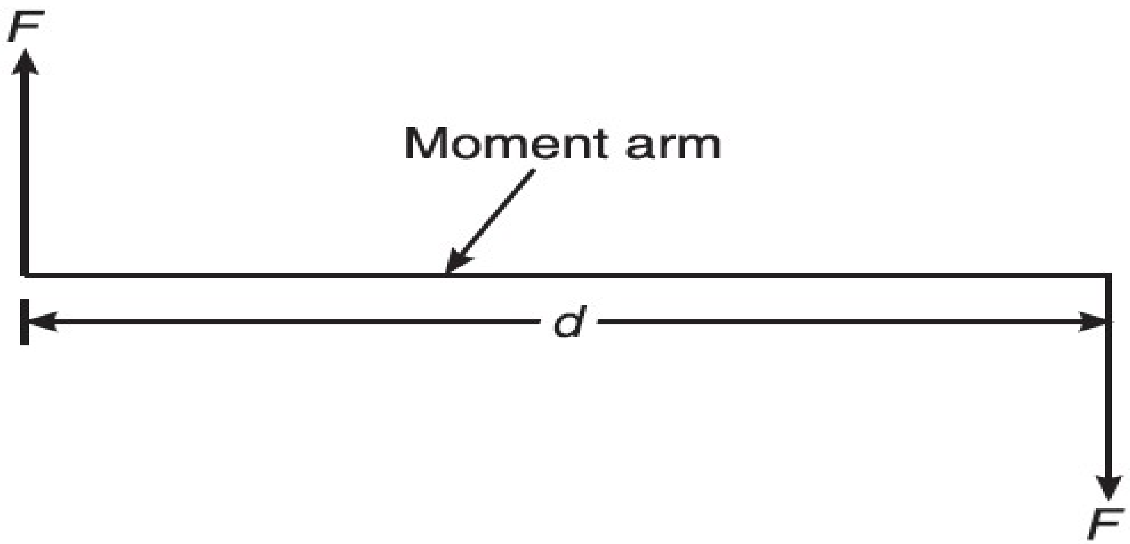
COUPLE

Two equal, opposite and parallel forces constitute a couple

Types of couple

(i) Clockwise couple

(ii) Anticlockwise couple



Properties of couple

(i) Two equal and opposite parallel forces are required to form a couple.

(ii) The magnitude of the moment of the couple = product of the magnitude of one of the forces and moment arm (perpendicular distance between the two forces).

(iii) Resultant of the forces of the couple is zero.

EQUIVALENT FORCE SYSTEM

It is a combination of a force passing through a given point and a moment about that point. The combination force is the resultant of all the forces acting on the body. And the moment is the sum of all the moment about that point.

Hence the Equivalent System consists of,

i. A Single Resultant force R passing through the point 'O'

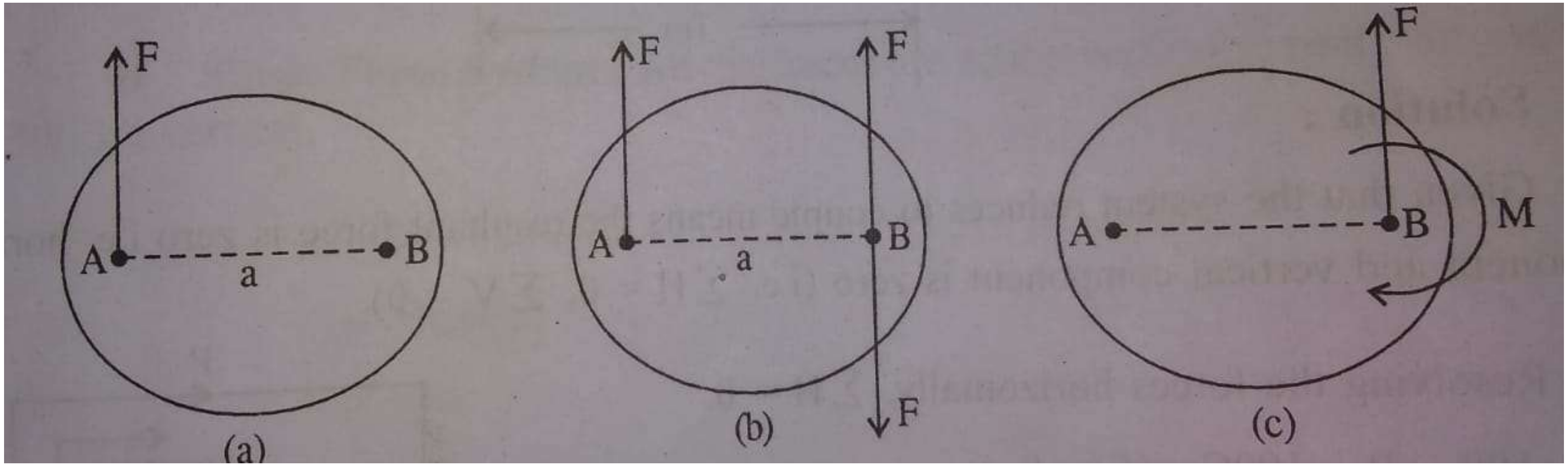
ii. Sum of all moments of all forces about the same point 'O'

RESOLUTION OF A FORCE INTO A FORCE AND A COUPLE

A given force F acting at a point A can be replaced by an equal force applied at another point B together with a Couple which will be equivalent to the original force.

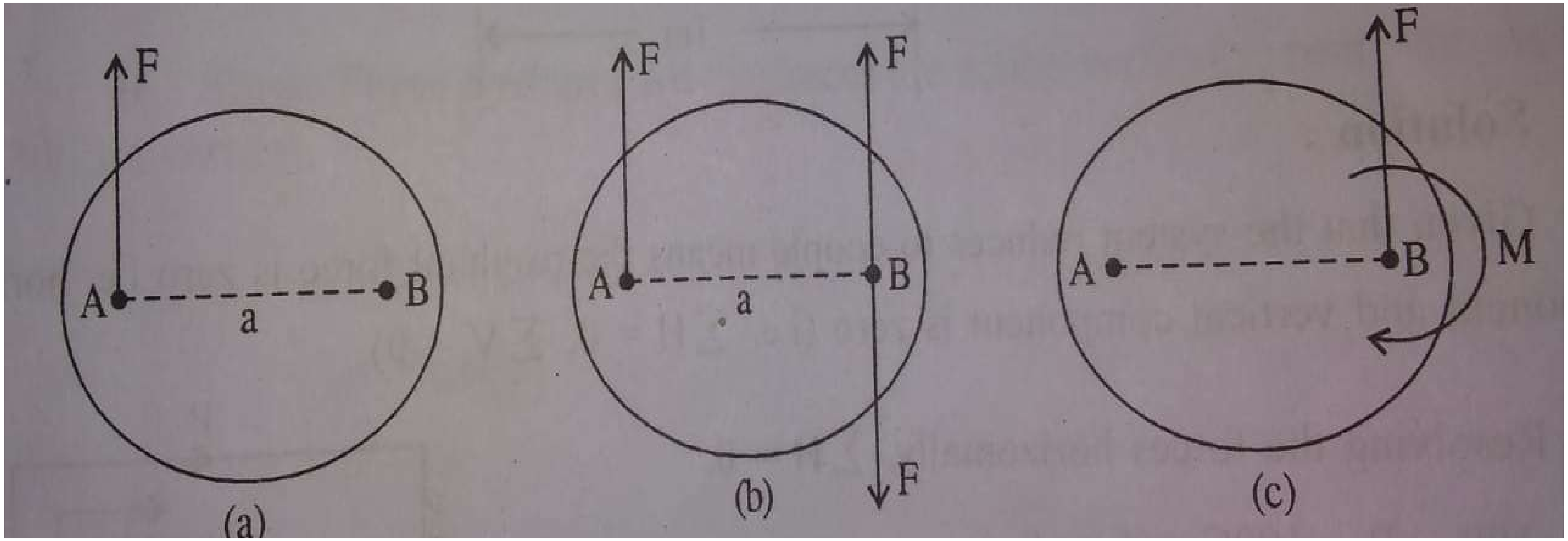
Figure shows a body subjected to a force F at A. Now it is required to shift this same force F at B. To do this follow the steps:

i. Keeping force F at A, superimpose another two equal opposite and collinear forces at B fig (b).



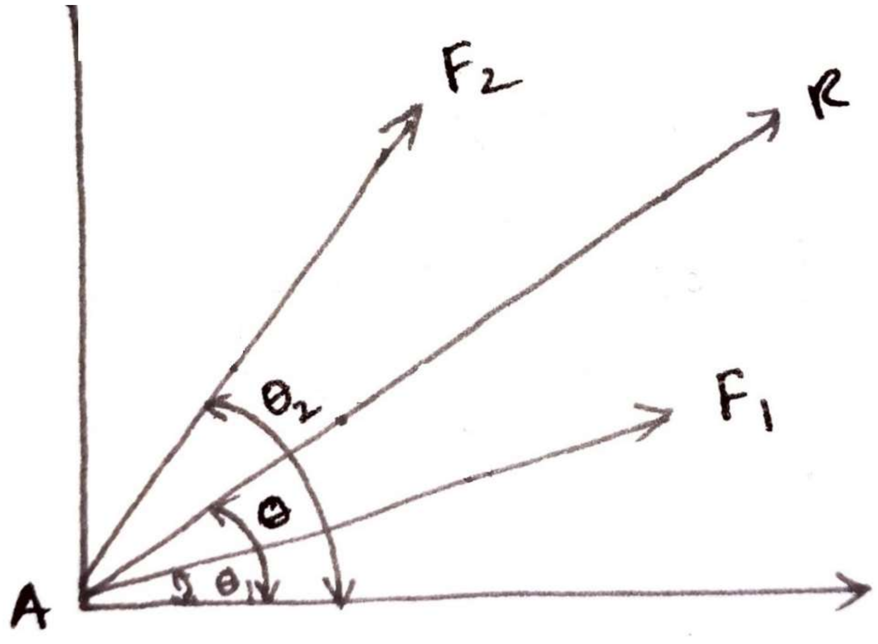
i. Now the force at A and one downward force at B will constitute a Couple rotating clockwise.

ii. Fig (c) shows the force F shifted at point B along with a couple M at B.

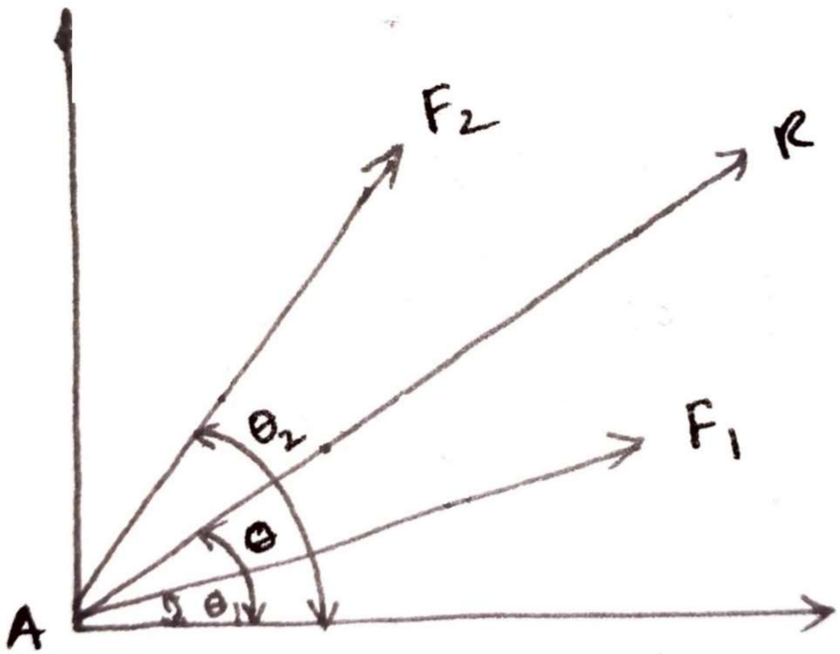


VARIGNON'S THEOREM OF MOMENTS

- This is also known as the principle of moments.
- The theorem states that "the algebraic sum of the moments of individual forces of a force system about a point is equal to the moment of their resultant about the same point".

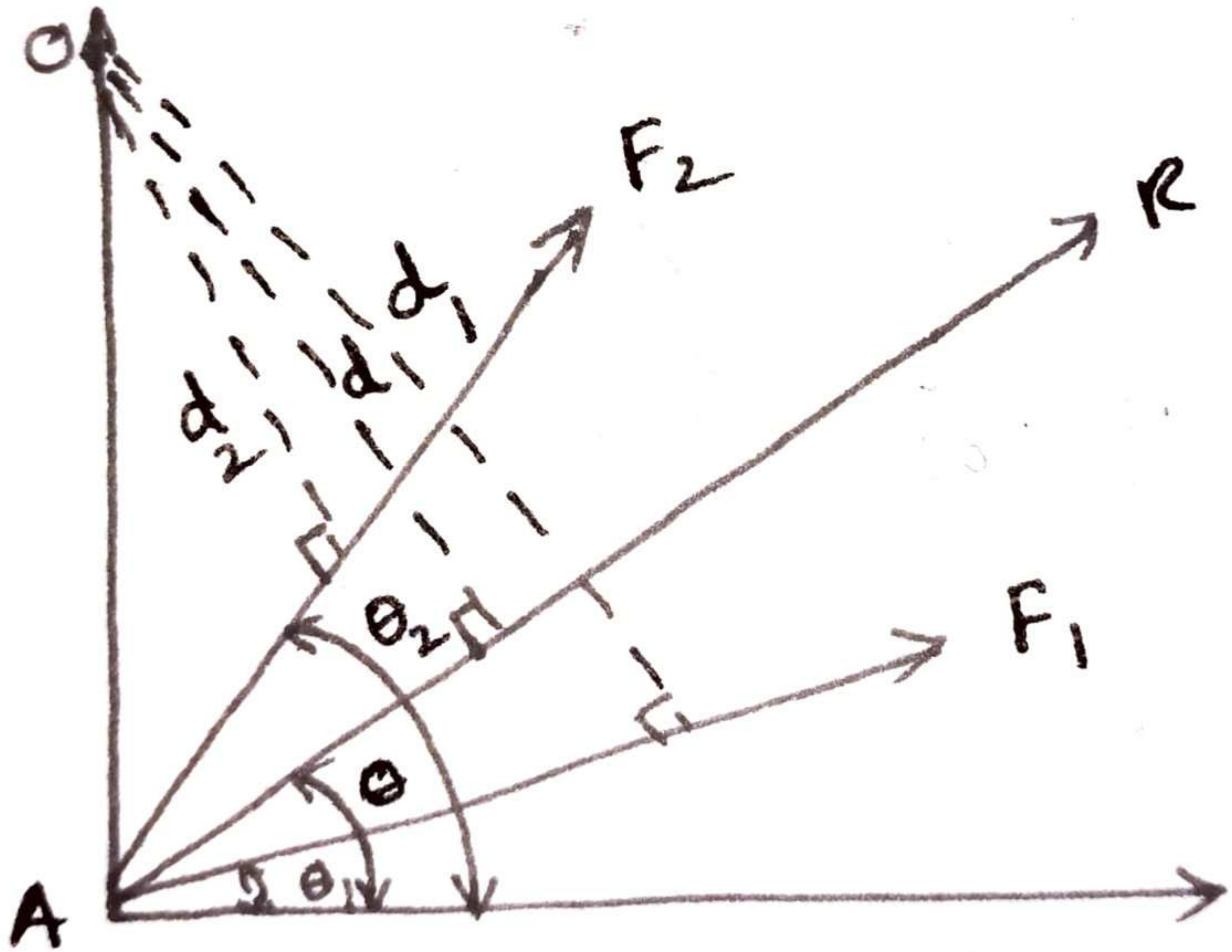


VARIGNON'S THEOREM OF MOMENTS



$$R_d = F_1d_1 + F_2d_2$$

VARIGNON'S THEOREM OF MOMENTS



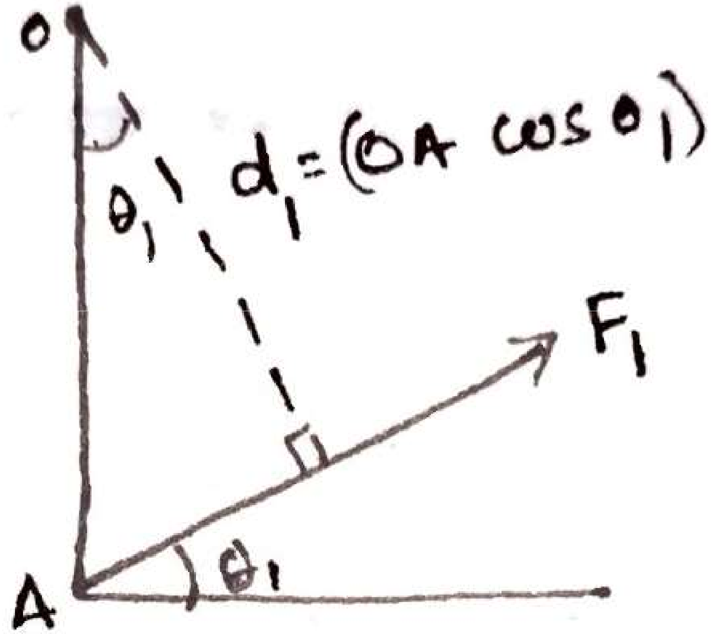
VARIGNON'S THEOREM OF MOMENTS

$$\Sigma M_O = F_1 d_1$$

$$F_1 d_1 = F_1 (OA \cos\theta_1)$$

$$F_1 d_1 = OA (F_1 \cos\theta_1)$$

$$F_1 d_1 = OA\, F_{1x} \longrightarrow (1)$$



VARIGNON'S THEOREM OF MOMENTS

III'ly

$$\Sigma M_O = F_2 d_2$$

$$F_2 d_2 = F_2 (OA \cos\theta_2)$$

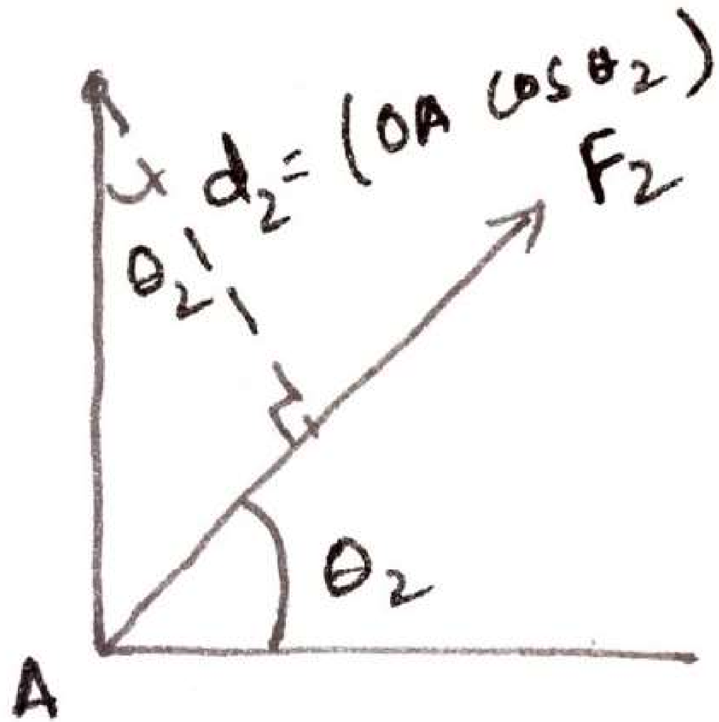
$$F_2 d_2 = OA (F_2 \cos\theta_2)$$

$$F_2 d_2 = OA\, F_{2x} \longrightarrow (2)$$

Combing equation (1) & (2)

$$F_1 d_1 + F_2 d_2 = OA (F_{1x} + F_{2x})$$

$$F_1 d_1 + F_2 d_2 = OA\, R_x \rightarrow (3)$$



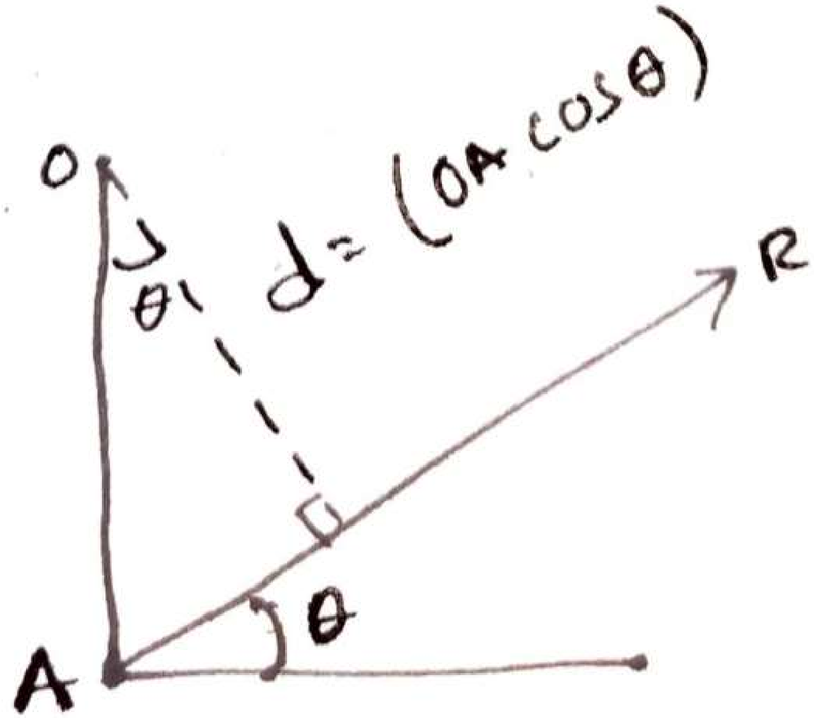
VARIGNON'S THEOREM OF MOMENTS

$$\sum M_o = Rd$$

$$Rd = R\,(OA\cos\theta)$$

$$Rd = OA\,(R\cos\theta)$$

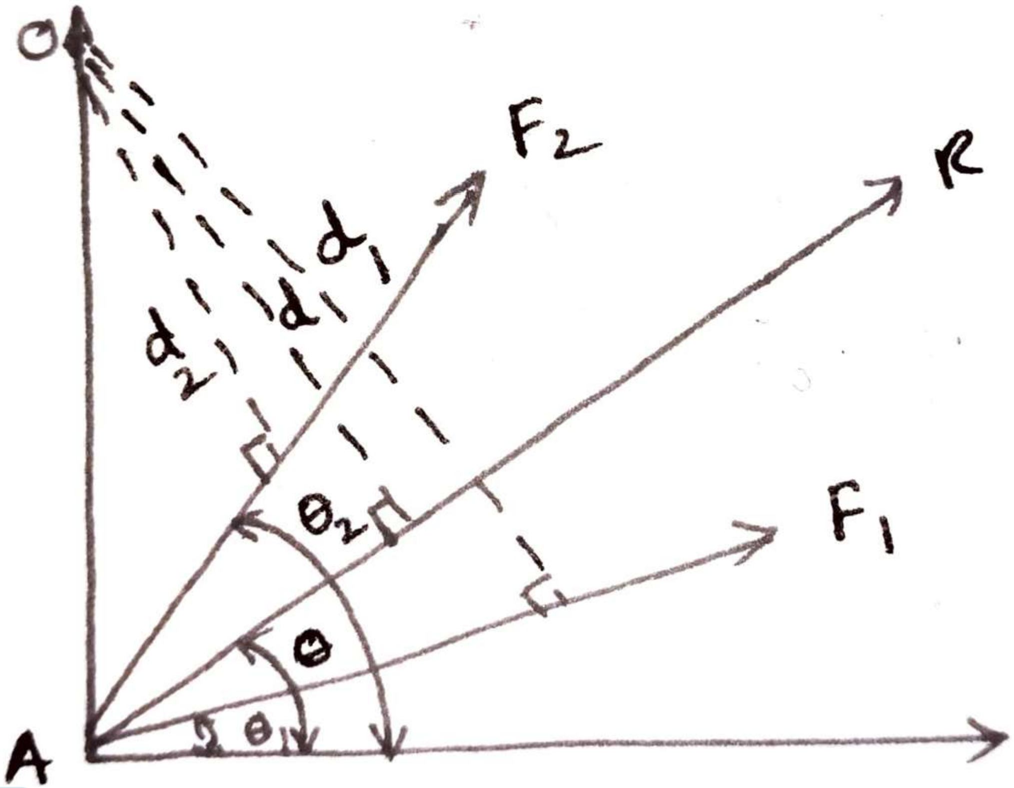
$$Rd = OA\,R_x \quad \text{--- (4)}$$



VARIGNON'S THEOREM OF MOMENTS

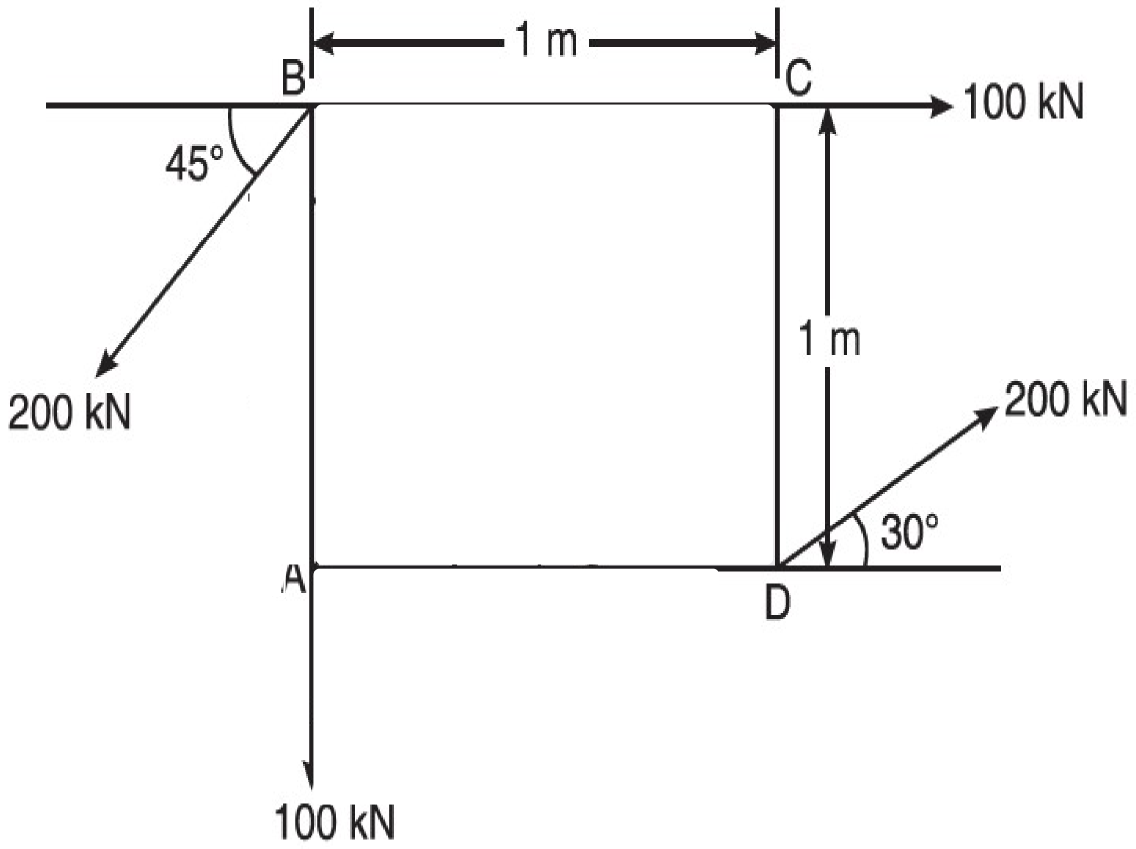
Comparing the Equation 3 and 4, we can write as

$$R_d = F_1 d_1 + F_2 d_2$$



NUMERICALS

1) A rigid plate ABCD is subjected to forces as shown in Figure. Compute the magnitude, direction and line of action of the resultant of the system with reference to the pc



NUMERICALS

1) A rigid plate ABCD is subjected to forces as shown in Figure. Compute the magnitude, direction and line of action of the resultant of the system with reference to the point A.

$$\Sigma F_x = 100 + 200 \cos 30° - 200 \cos 45° = 131.78 \text{ kN}$$

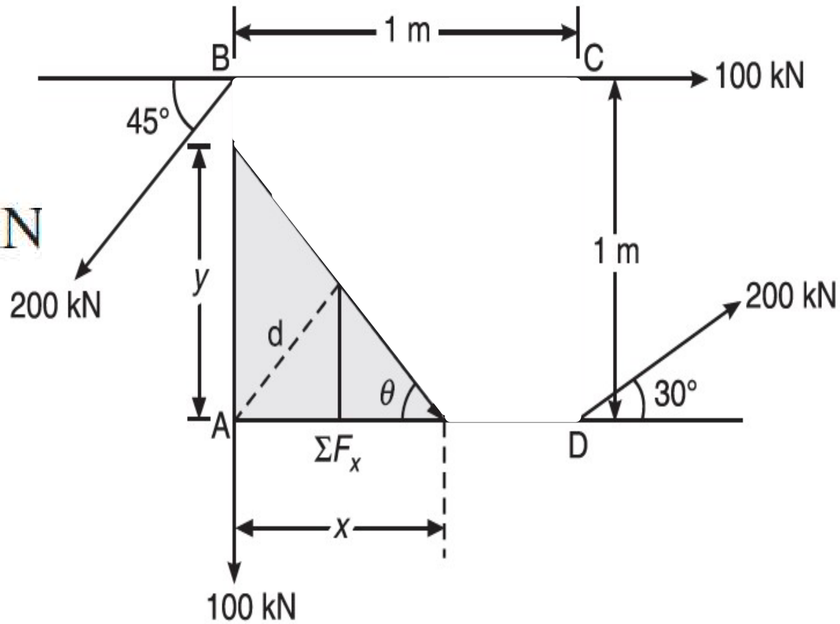
$$\Sigma F_y = -100 - 200 \sin 45° - 200 \sin 30° = -141.42 \text{ kN}$$

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$$

$$R = \sqrt{(131.78)^2 + (-141.42)^2} = 193.30 \text{ kN}$$

$$\tan \theta = \frac{\Sigma F_y}{\Sigma F_x} = \frac{-141.42}{131.78} = -1.073$$

$$\theta = \tan^{-1}(-1.073) = -42.02°$$



The line of action of 100 kN is directly passing through the point A, therefore, the moment produced by the 100 kN force about A is zero.

$$\Sigma M_A = 100 \times 0 + 200 \sin 45° \times 0 - 200 \cos 45° \times 1$$

$$+ 100 \times 1 - 200 \sin 30° \times 1 + 200 \cos 30° \times 0$$

$$= -200 \cos 45° = -141.42 \text{ kN} - \text{m}$$

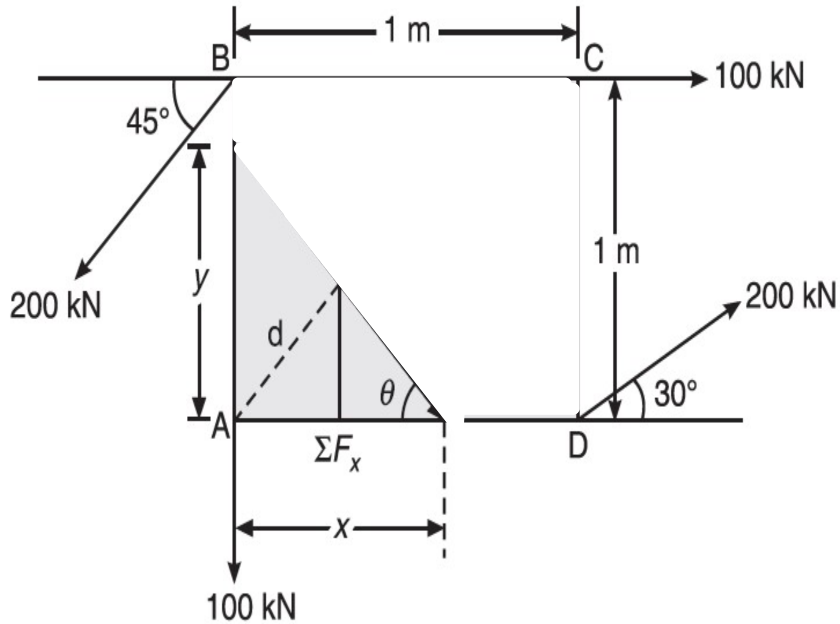
$$d = \frac{\Sigma M_A}{R} = \left| \frac{-141.42}{193.3} \right| = 0.732 \text{ m}$$

$$x\text{-intercept} = \left| \frac{\Sigma M_A}{\Sigma F_y} \right|$$

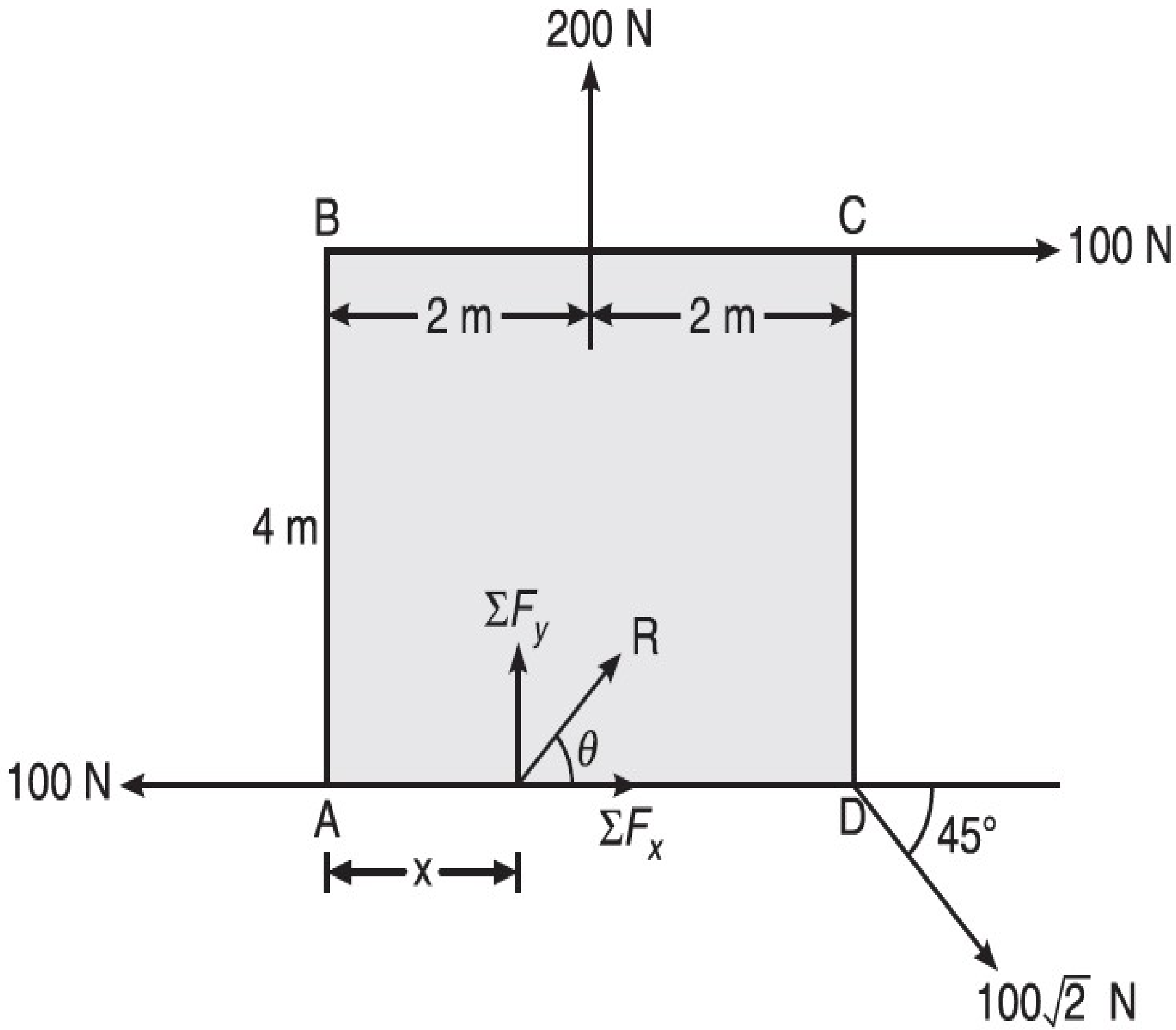
$$= \left| \frac{-141.42}{-141.42} \right| = 1 \text{ m}$$

$$y\text{-intercept} = \left| \frac{-141.42}{131.78} \right|$$

$$= 1.073 \text{ m}$$



2) For the non-concurrent coplanar system shown in Figure, determine the magnitude, direction and position of the resultant force with reference to A.



2) For the non-concurrent coplanar system shown in Figure, determine the magnitude, direction and position of the resultant force with reference to A.

$$\Sigma F_x = 100 - 100 + 100\sqrt{2}\cos 45^\circ + 0 = 100 \text{ N}$$

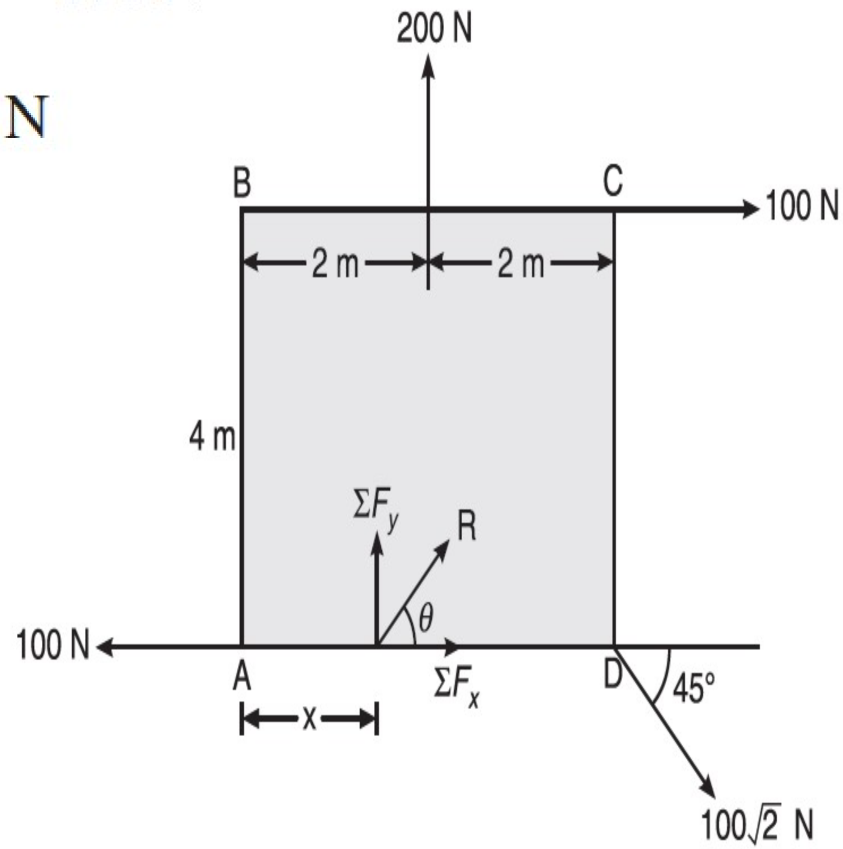
$$\Sigma F_y = 200 - 100\sqrt{2}\sin 45^\circ + 0 = 100 \text{ N}$$

$$R = \sqrt{(100)^2 + (100)^2}$$

$$= 100\sqrt{2} \text{ N} = 141.421 \text{ N}$$

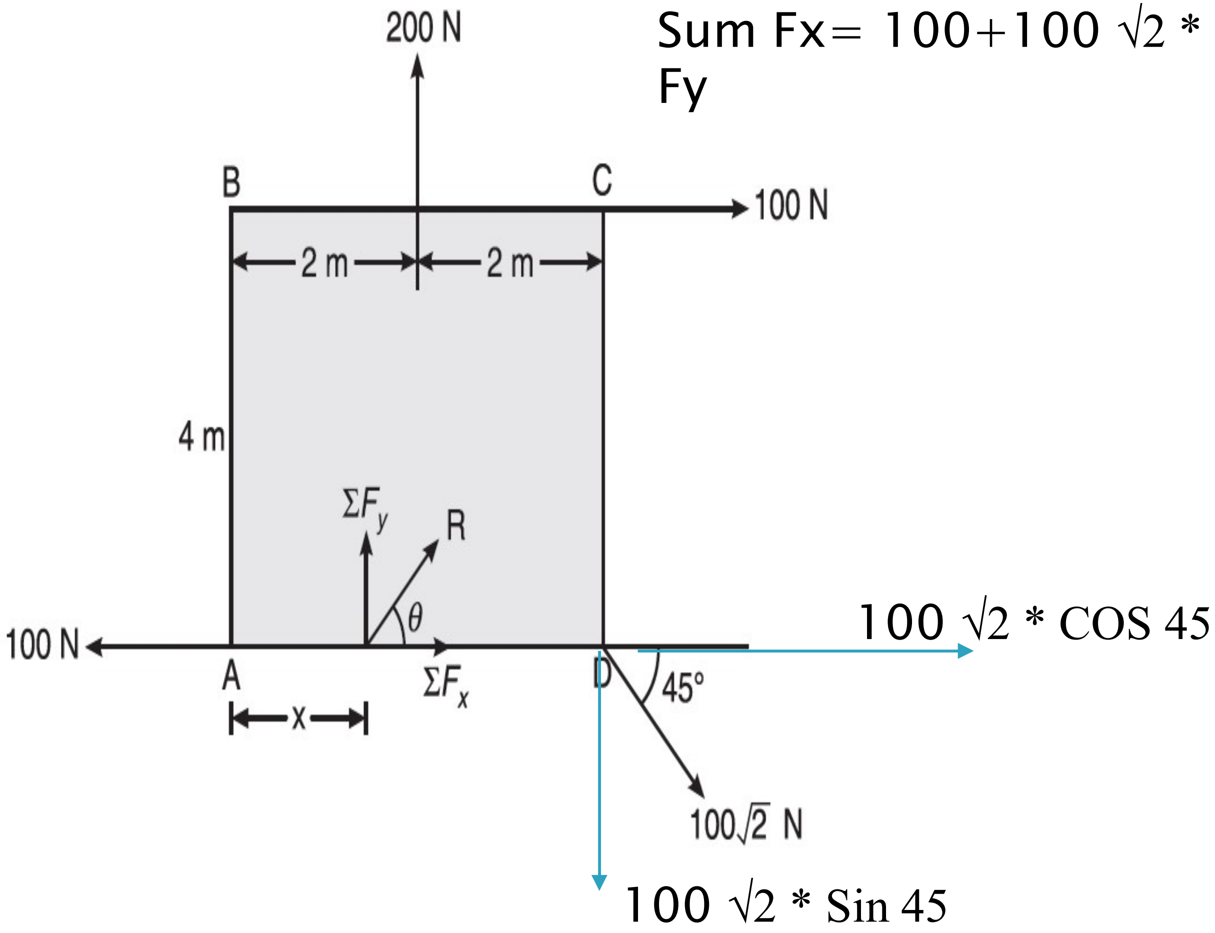
$$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x} = \frac{100}{100} = 1$$

$$\theta = 45^\circ$$



Sum Fx= 100+100 √2 * COS 45-100= 100

Fy

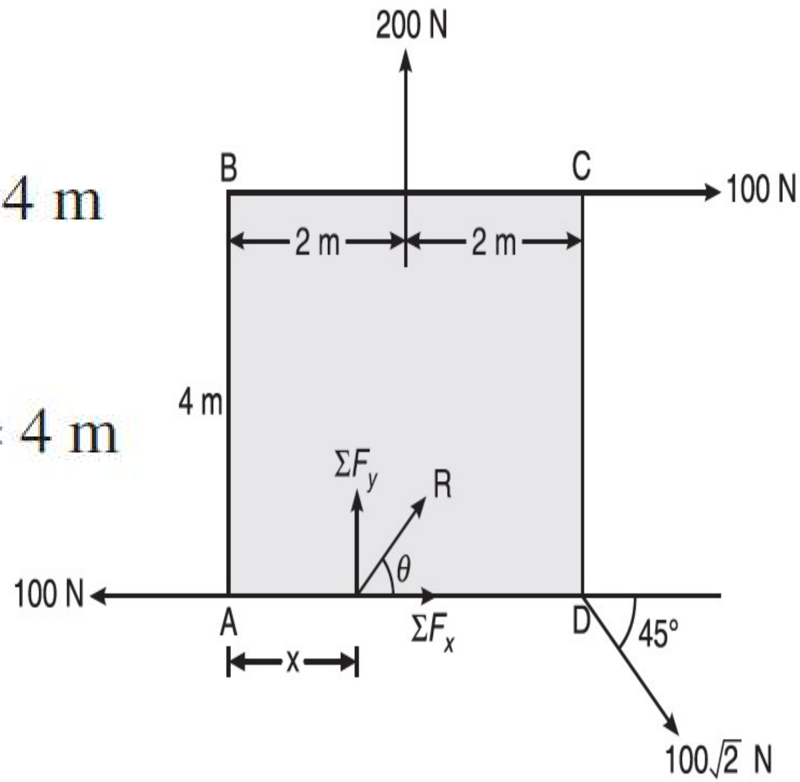


$$\Sigma M_A = 100 \times 0 + 100\sqrt{2}\,\sin 45° \times 4 + 100 \times 4 - 200 \times 2$$
$$= 100\sqrt{2} \times \sin 45° \times 4 = 400 \text{ N-m}$$

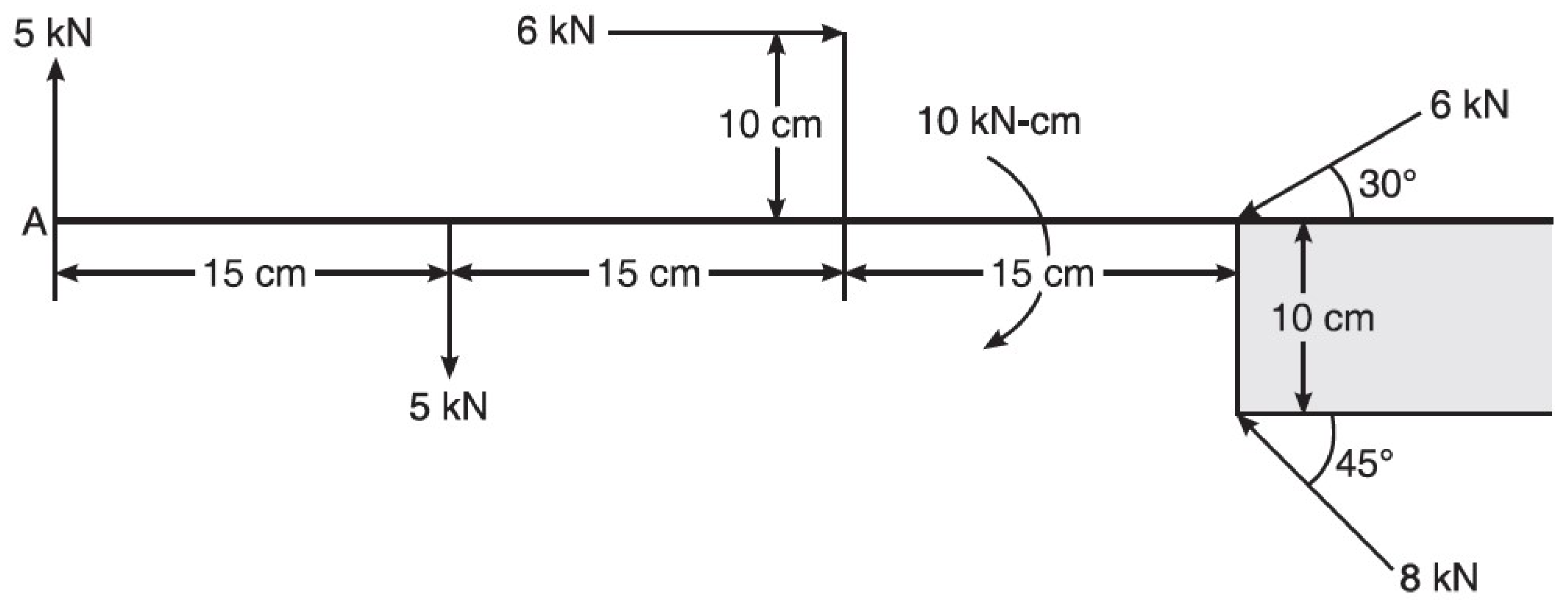
$$d = \frac{400}{100\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2} = 2.82 \text{ m}$$

$$x\text{-intercept} = \left|\frac{\Sigma M_A}{\Sigma F_y}\right| = \left|\frac{400}{100}\right| = 4 \text{ m}$$

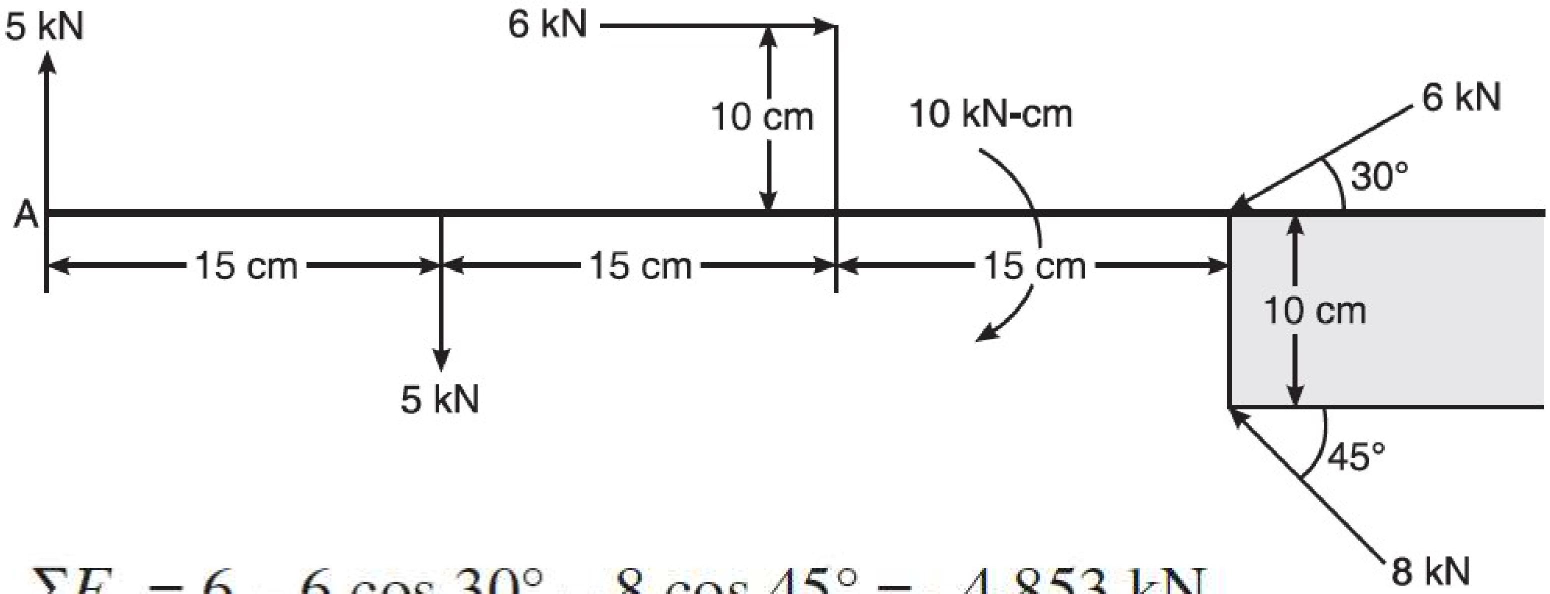
$$y\text{-intercept} = \left|\frac{\Sigma M_A}{\Sigma F_x}\right| = \left|\frac{400}{100}\right| = 4 \text{ m}$$



3) A bracket is subjected to five forces and a couple as shown in Figure. Determine the magnitude, direction and the line of action of the resultant.



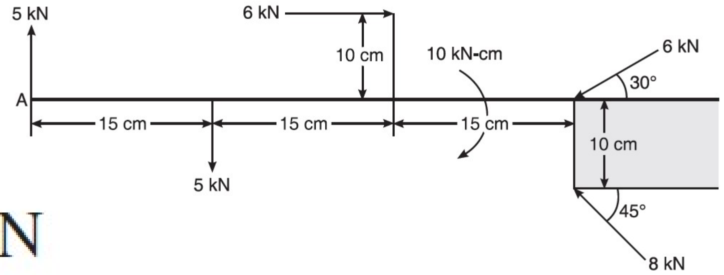
3) A bracket is subjected to five forces and a couple as shown in Figure. Determine the magnitude, direction and the line of action of the resultant.



$$\Sigma F_x = 6 - 6\cos 30° - 8\cos 45° = -4.853 \text{ kN}$$

$$\Sigma F_y = 5 - 6\sin 30° + 8\sin 45° - 5 = 2.656 \text{ kN}$$

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$$



$$R = \sqrt{(-4.853)^2 + (2.656)^2} = 5.532 \text{ kN}$$

$$\theta = \tan^{-1} \frac{\Sigma F_y}{\Sigma F_x}$$

$$\theta = \tan^{-1} \left(\frac{2.656}{-4.853} \right) = -28.69°$$

Line of action of the resultant: Here we can take moment about any point

$$d = \frac{\Sigma M_A}{R}$$

$$\Sigma M_A = 5 \times 15 + 6 \times 10 + 6 \sin 30° \times 45 + 8 \cos 45° \times 10 - 8 \sin 45° \times 45 + 10$$
$$= 82 \text{ kN-cm}$$

$$d = \left| \frac{82}{5.532} \right| = 14.825 \text{ cm}$$

$$x\text{-intercept} = \left| \frac{\Sigma M_A}{F_y} \right| = \left| \frac{82}{2.656} \right| = 30.873 \text{ cm}$$

$$y\text{-intercept} = \left| \frac{\Sigma M_A}{F_x} \right| = \left| \frac{82}{-4.853} \right| = 16.897 \text{ cm}$$

FREE BODY DIAGRAM (FBD)

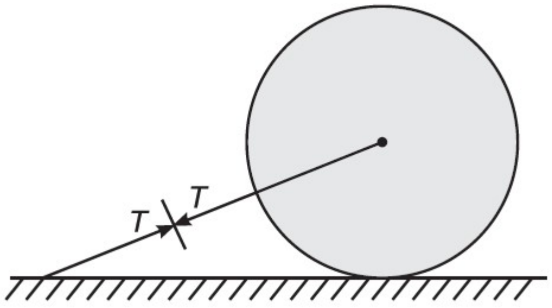
- Free-body diagram is the diagram which represents the various forces acting on the body.
- For the analysis of equilibrium condition it is necessary to isolate the body under consideration from other bodies in contact and draw all forces acting on the other body.

FREE BODY DIAGRAM (FBD)

- The type of diagram of the body in which the body under consideration is free from all contact surfaces and is shown with all the forces on it (including self weight, reactions and applied forces) is called Free Body Diagram (FBD).

COMPREHENSIVE QUESTIONS

Let us consider a spherical ball of mass m, placed on a horizontal plane and tied to the plane by a string as shown in Figure (a).



(a)

1) Draw FBD

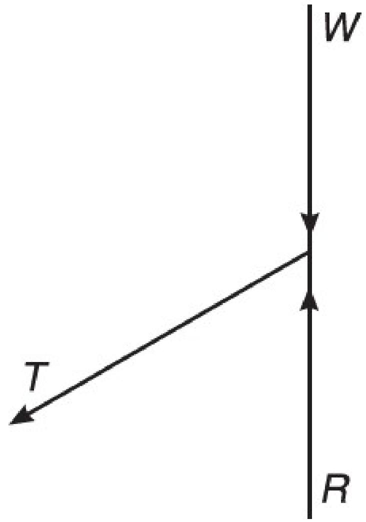
COMPREHENSIVE QUESTIONS

Figure shows the free-body diagram of the spherical ball subjected to various forces like:

(i) Self weight, *W*, always acting vertically downwards.

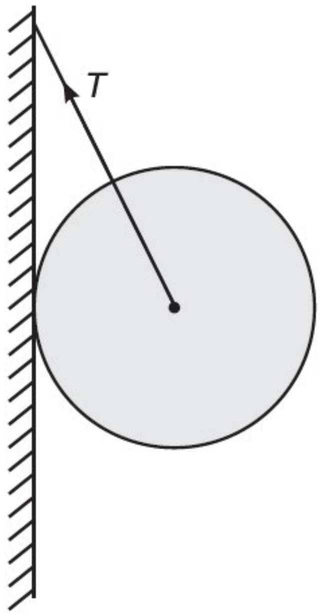
(ii) Normal reaction, *R*, always acting perpendicular to the plane under consideration.

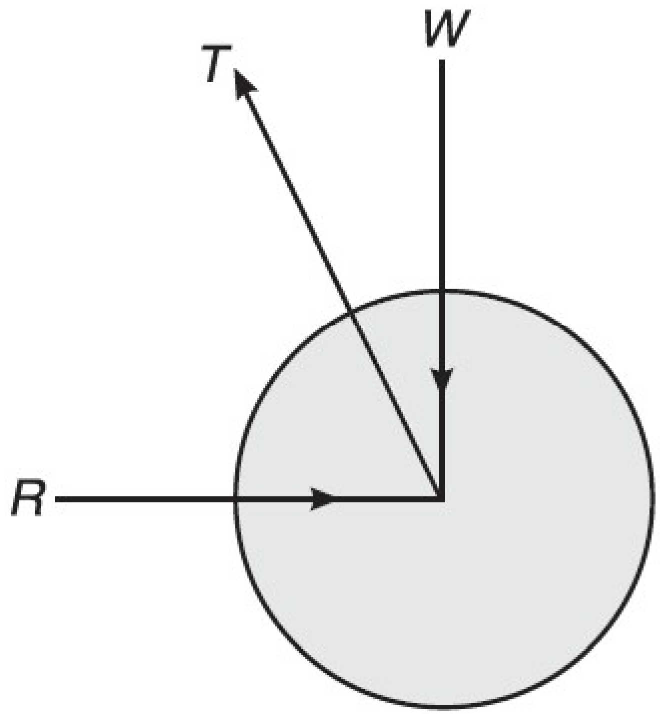
(iii) Tension *T* in the string



COMPREHENSIVE QUESTIONS

Draw FBD a spherical ball supported by a string and resting against a wall





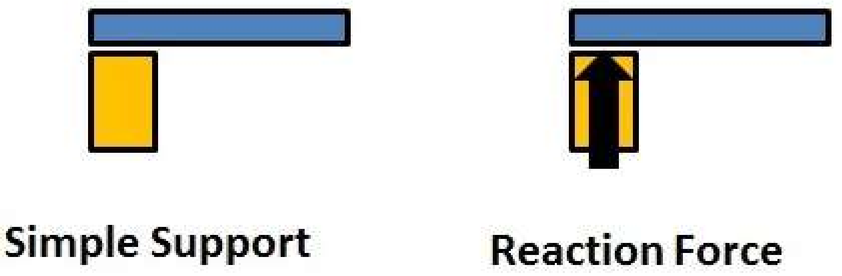
SUPPORT

Supports are structures which prevent the beam or the body from moving and help to maintain equilibrium.

<u>Types of Supports</u>

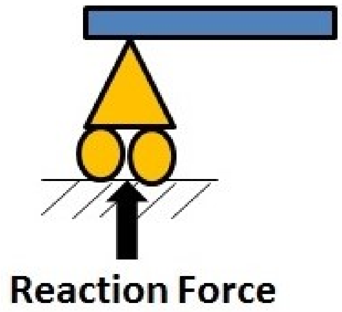
1. Simple support

- This is a support where a beam rests freely on a support.
- The beam is free to move only horizontally and also can rotate about the support.
- In such a support one reaction, which is perpendicular to the plane of support, is developed.

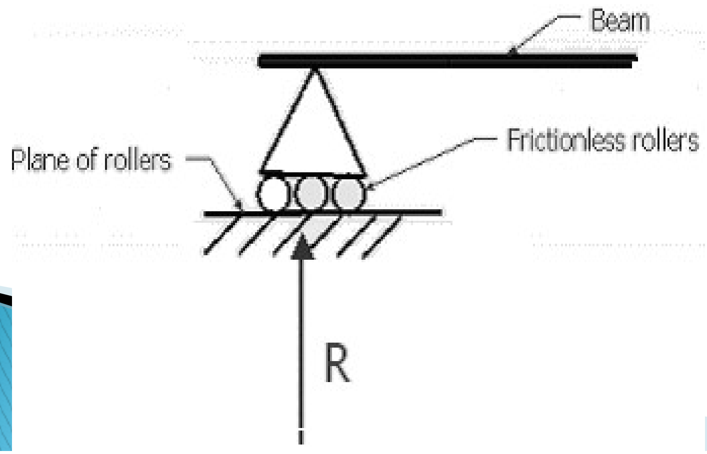


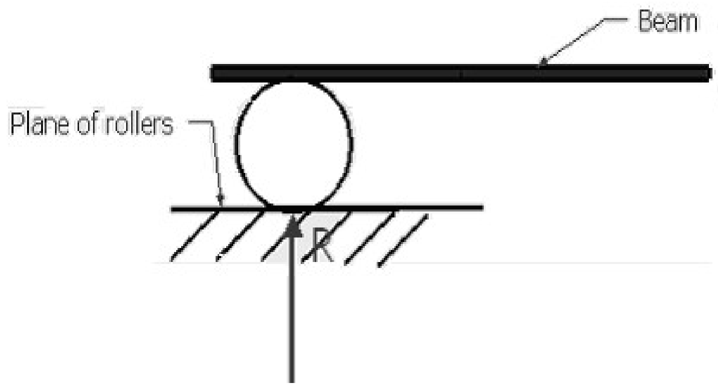
Types of Supports

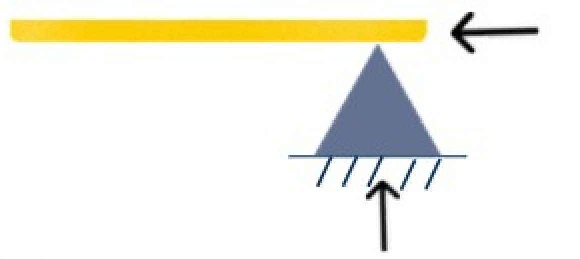
2. Roller support



- This is a support in which a beam rests on rollers, which are frictionless.
- At such a support, the beam is free to move horizontally and as well rotate about the support.
- Here one reaction which is perpendicular to the plane of rollers is developed



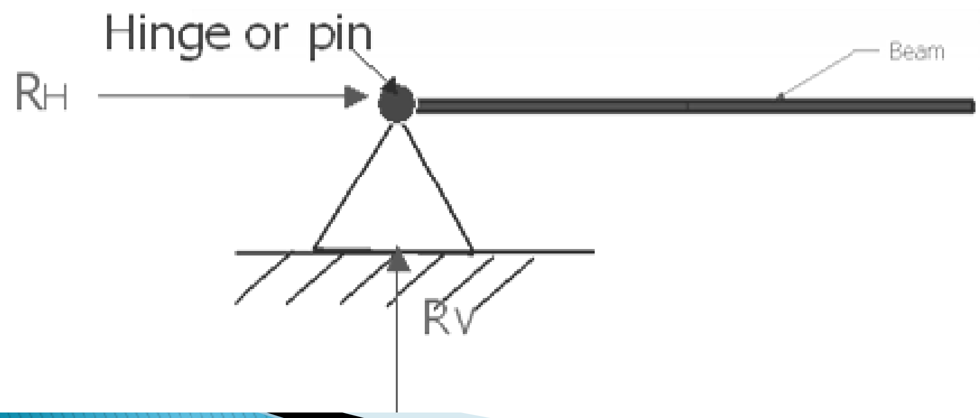




Types of Supports

3. Hinged support

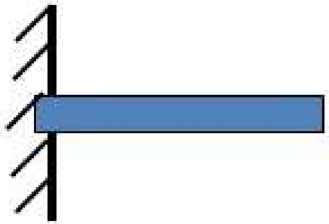
- This is a support in which the beam is attached to a support by means of a hinge or pin.
- The beam is not free to move in any direction but can rotate about the support.
- In such a support a horizontal reaction and a vertical reaction will develop.



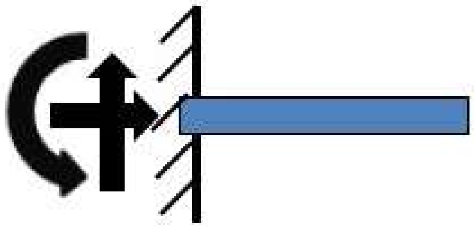
Types of Supports

4. Fixed support

- This is a support which prevents the beam from moving in any direction and also prevents rotation of the beam.
- In such a support a horizontal reaction, vertical reaction and a Fixed End Moment are developed to keep the beam in equilibrium.



Fixed Support

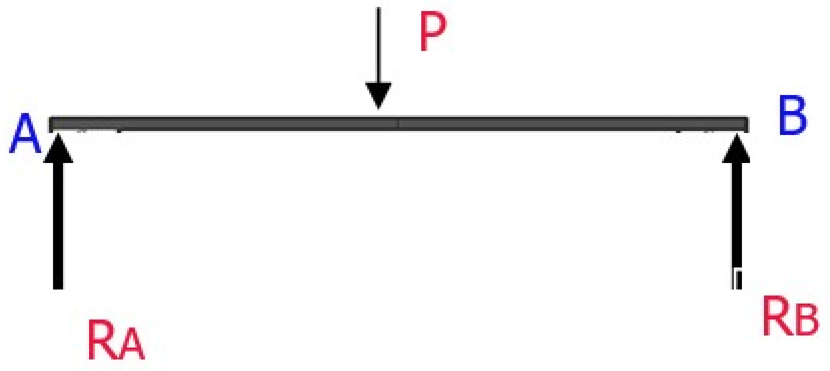


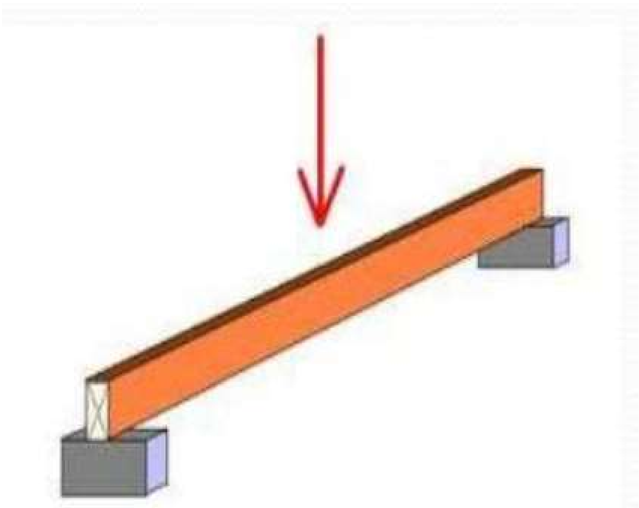
Reaction Force

LOADS

1. Point load or Concentrated load

- If a load acts over a very small length of the beam, it is assumed to act at the mid-point of the loaded length and such a loading is termed as Point load

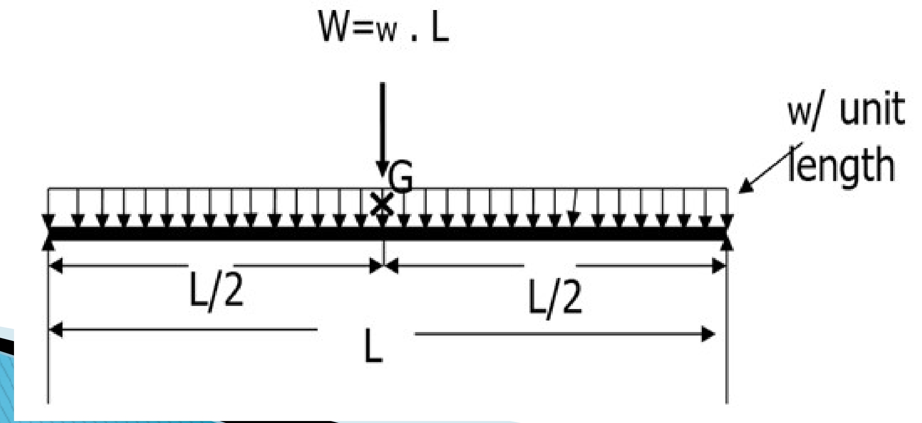


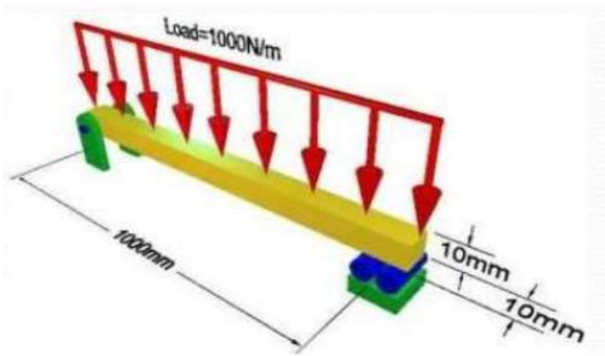


LOADS

2. Uniformly distributed load (UDL)

- If a beam is loaded in such a manner that each unit length of the beam carries the same intensity of loading, then such a loading is called UDL.
- The UDL should be replaced by an equivalent point load or total load acting through the mid point of the loaded length.

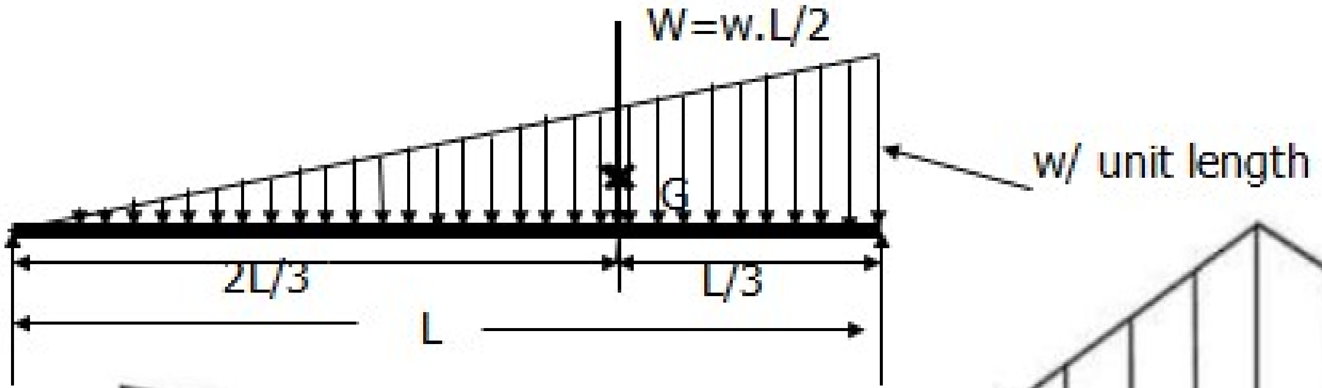


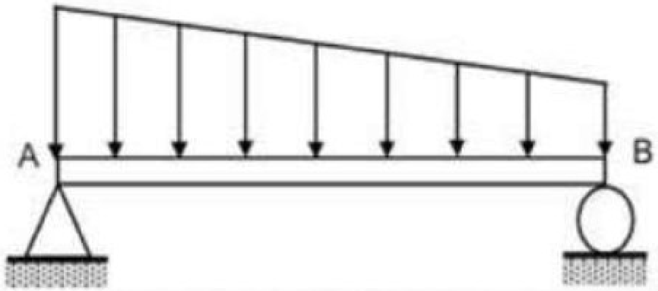


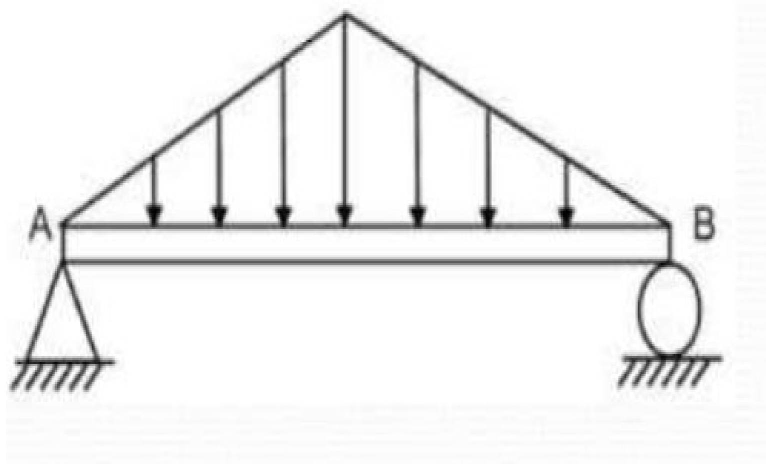
LOADS

3. Uniformly varying load (UVL)

- If a beam is loaded in such a manner, that the intensity of loading varies linearly or uniformly over each unit distance of the beam, then such a load is



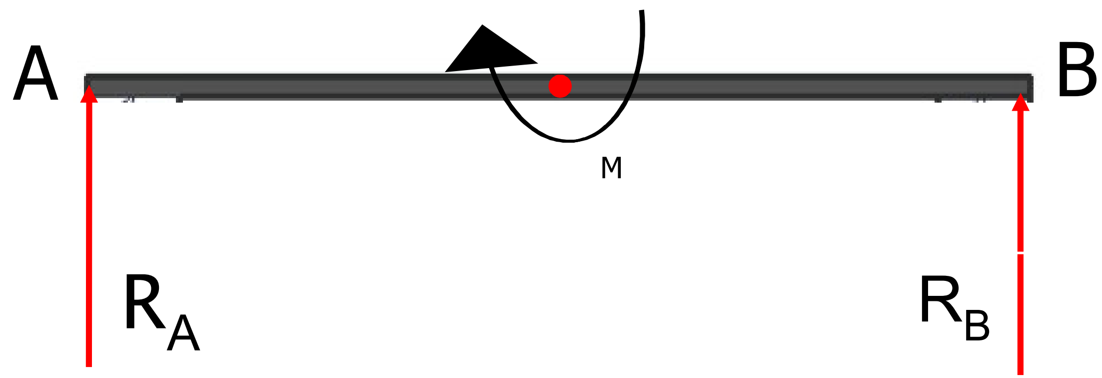




LOADS

4. External Moment

- A beam can also be subjected to external moments at certain points as shown in figure.
- These moments should be considered while calculating the algebraic sum of moments of forces about a point on the beam.



BEAMS

- It is a structural element that is capable of withstanding load primarily by resisting its bending forces.
- They are made of steel or reinforced concrete (RCC) or steel.

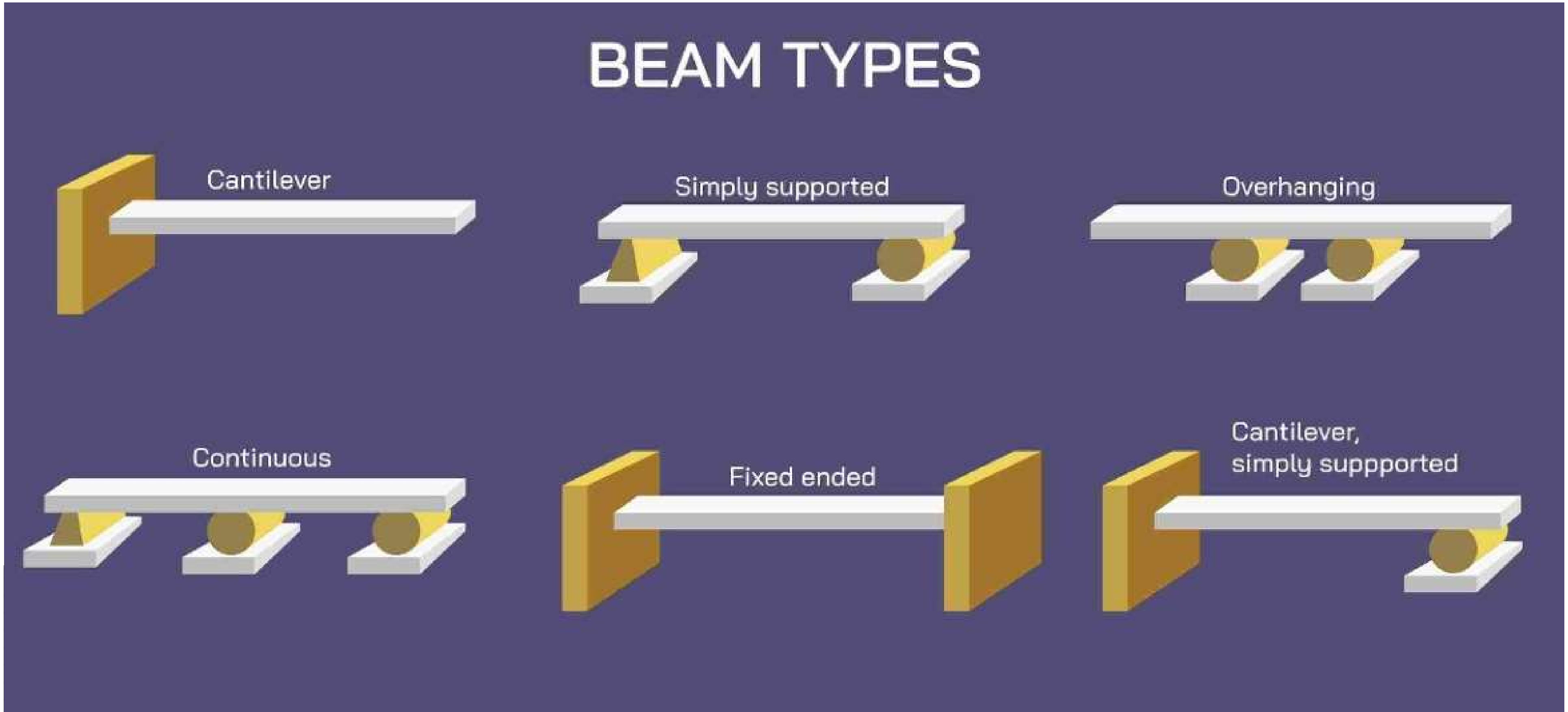
Beams are used in the structure to

1. Resist loads
2. Counter bending moment and shear forces.
3. Connect the frame.
4. Provide a uniform distribution of loads

BEAMS

Classification of beams:

According to the support conditions



BEAMS

Functions of Beam:

- The primary function of beam is to carry and transfer the loads imposed on a structure.
- Beams support the weight of floors, walls, roofs, and any other imposed loads such as furniture, equipment, or people. They distribute these loads to the columns.
- Beams prevent sagging, deflection, or excessive bending of the structural members they support.

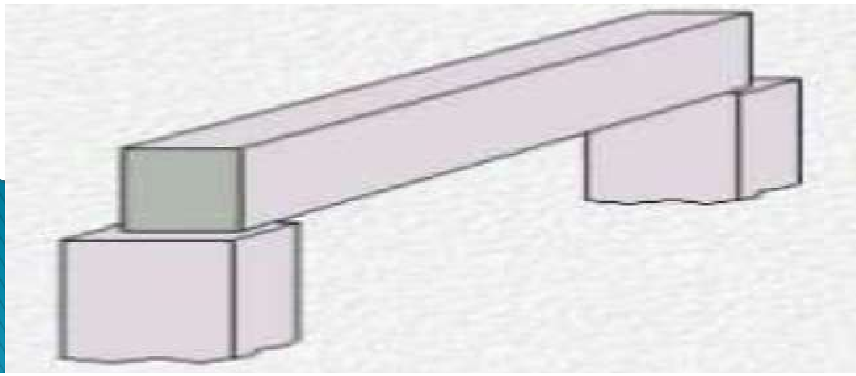
Types of beams

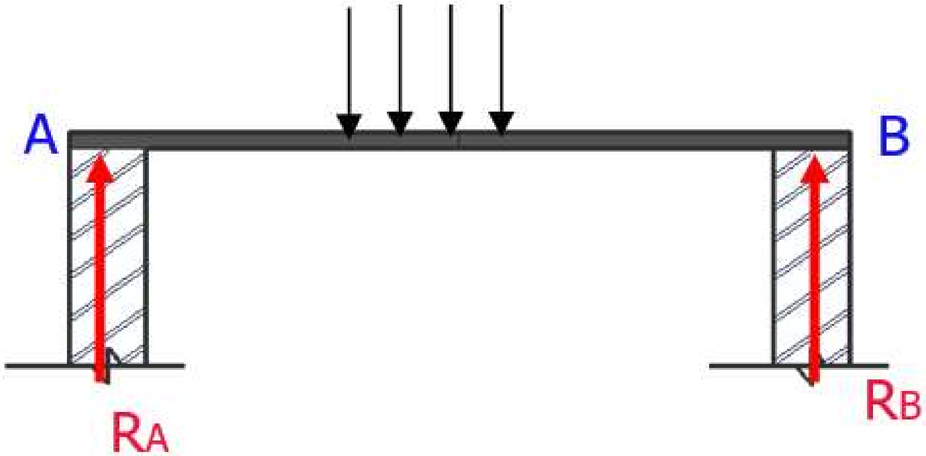
A) Statically Determinate Beams

The equations of static equilibrium are sufficient to find all the unknown reactions. The types of beams which are statically determinate are as follows:

1) Simply Supported Beams

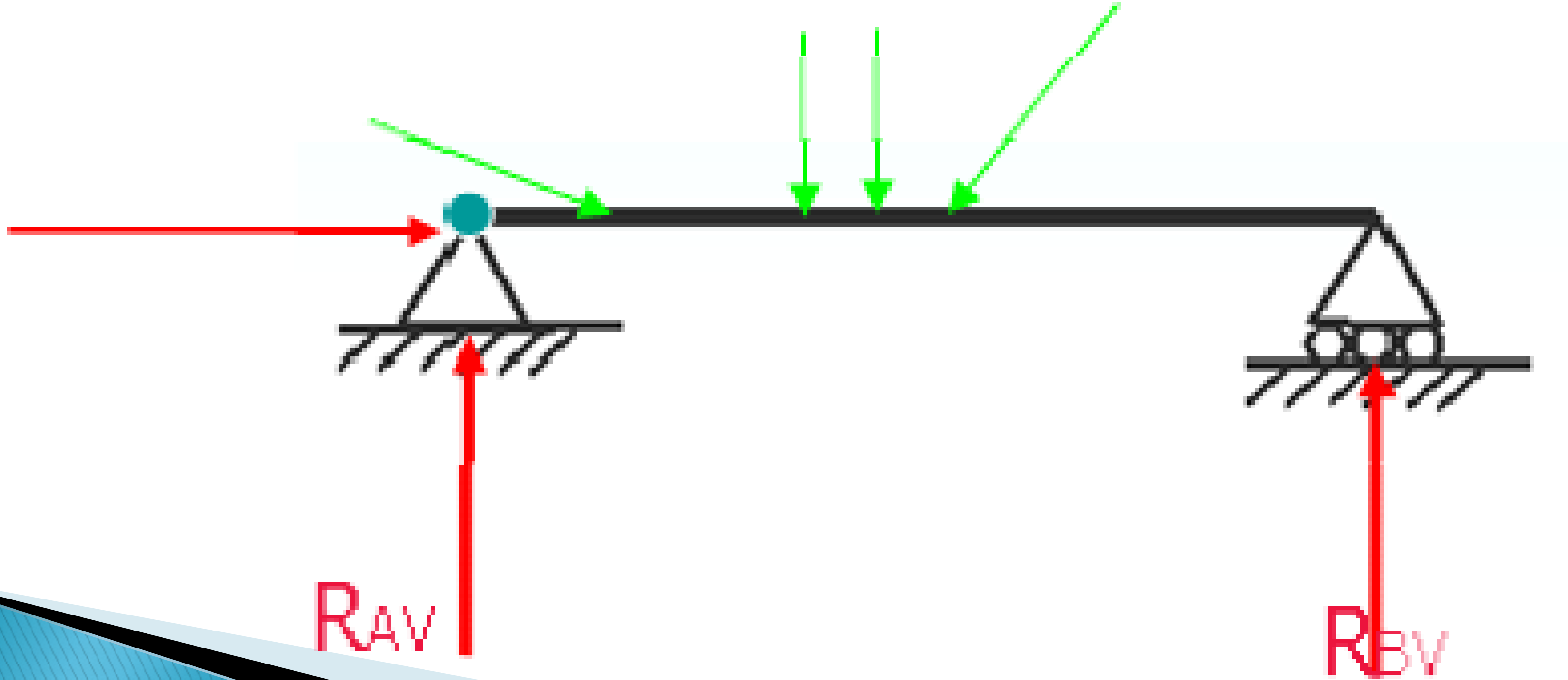
A beam is said to be simply supported when both ends of the beam rest on simple supports. Such a beam can carry or resist vertical loads only.





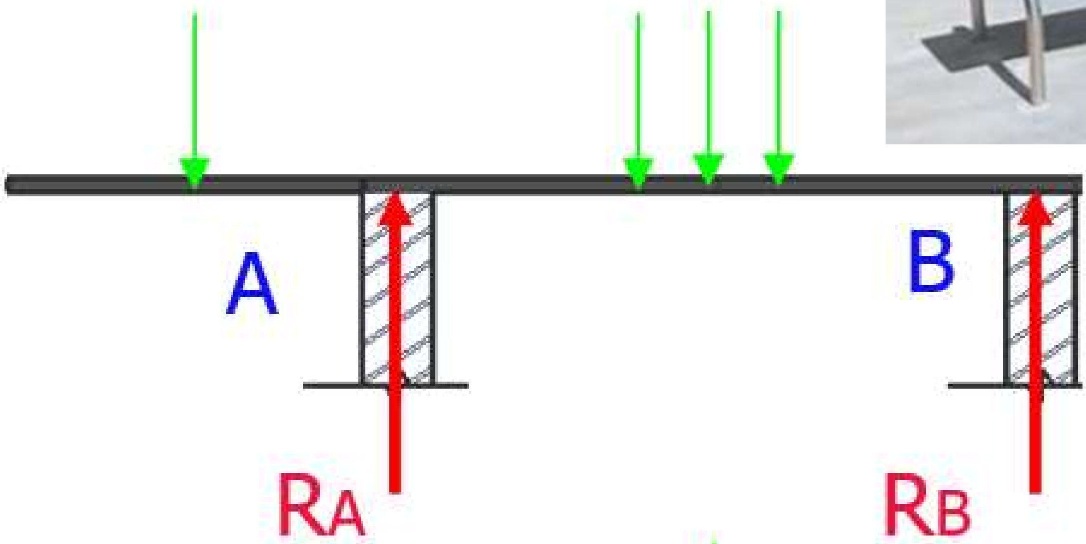
2) Beam with one end hinged & other on rollers

It is a beam where one end of the beam is hinged to a support and the other end rests on a roller support. Such a beam can carry any type of loads.



3) Over hanging beam

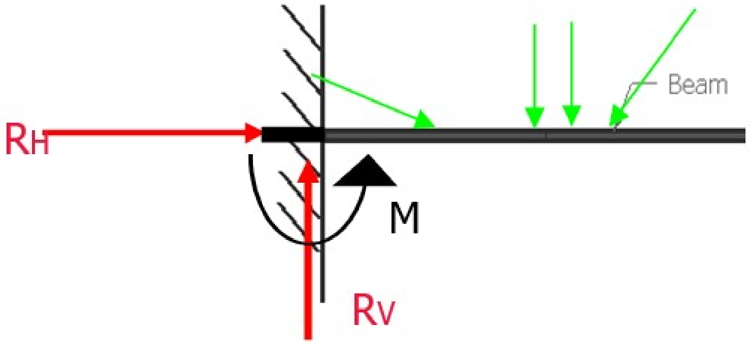
It is a beam which projects beyond the supports. A beam can have over hanging portions on one side or on both sides.

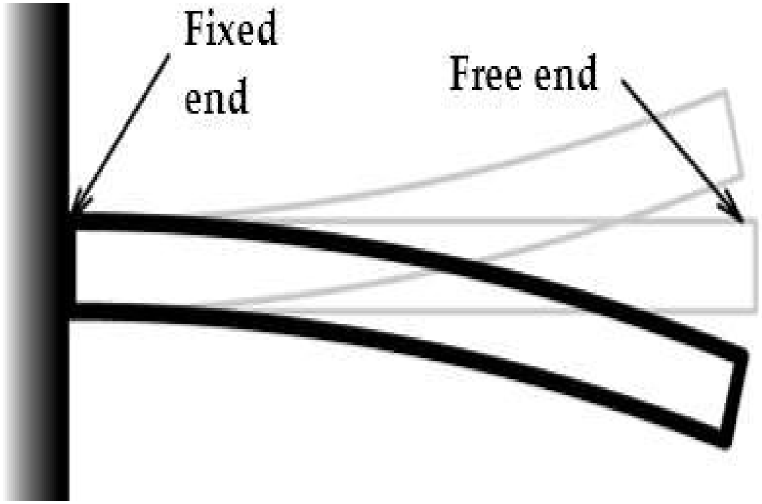


4) Cantilever Beams

It is a beam, with one end fixed and other and free.

Such a beam can carry loads in any directions.



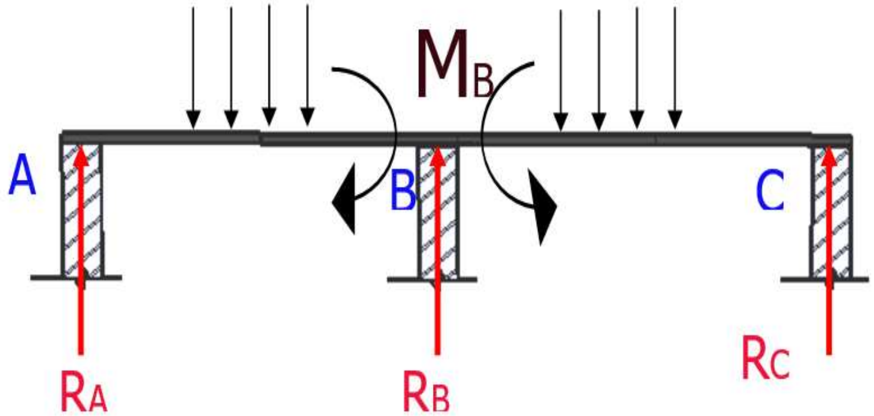


B) Statically Indeterminate Beams

In statically indeterminate beams, the number of unknown reactions exceeds the number of equations of static equilibrium and hence cannot be solved completely using these equations alone.

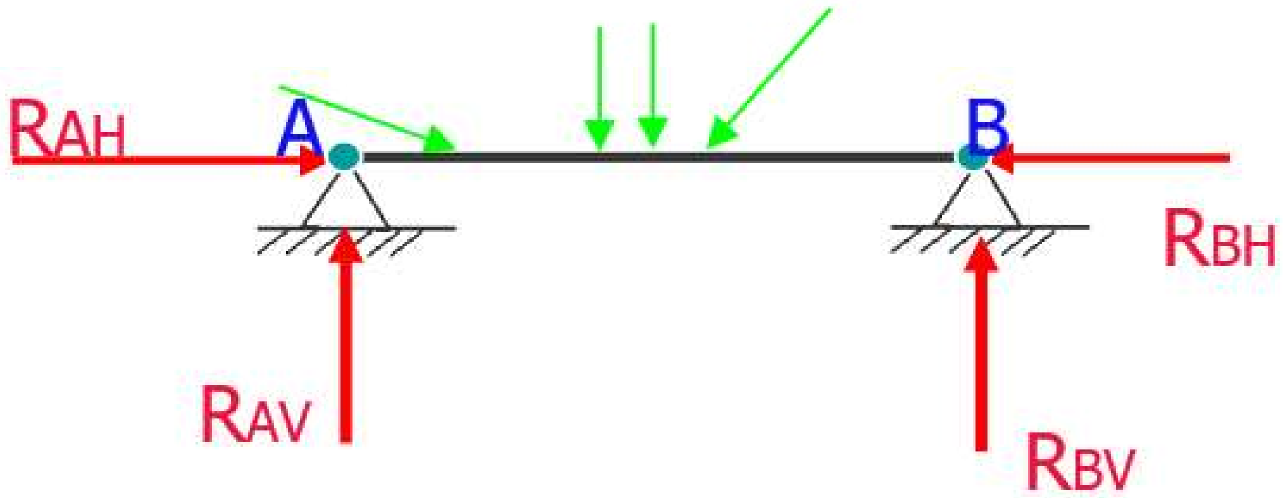
1) Continuous Beams

It is a beam which rests over a series of supports at more than two points.



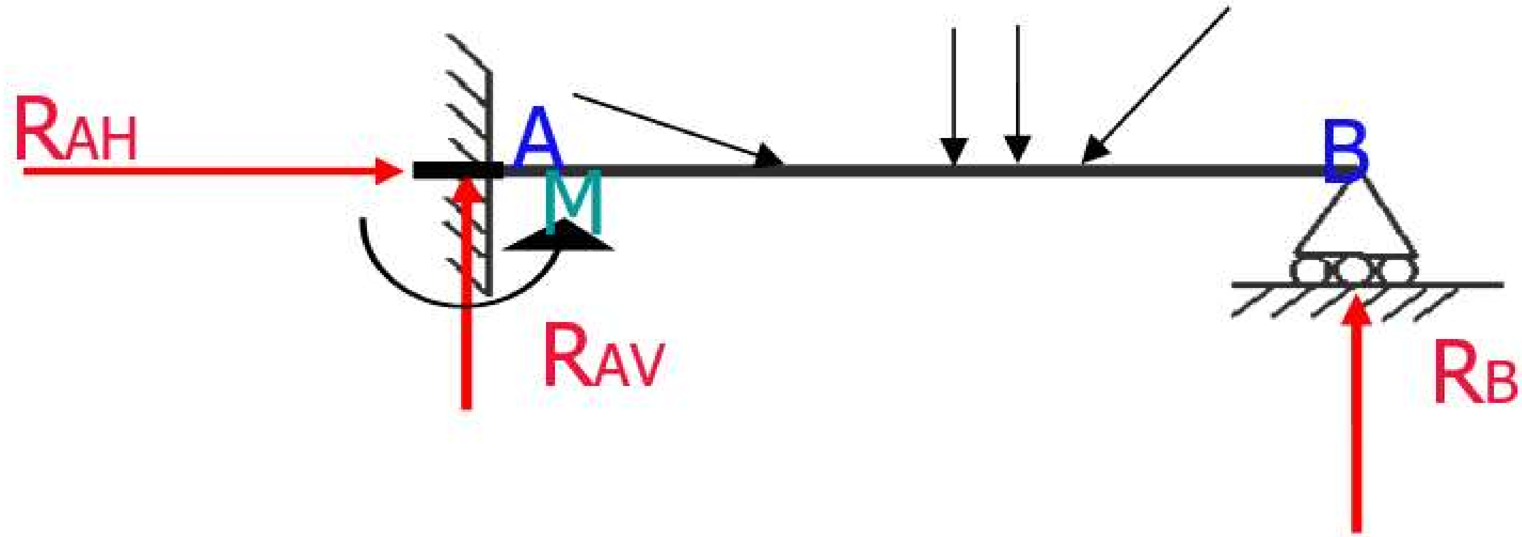
2) Hinged Beam (Beams with both ends hinged)

It is a beam which is hinged to supports at both ends. Such a beam can carry loads is any direction.



3) Propped Cantilever Beam

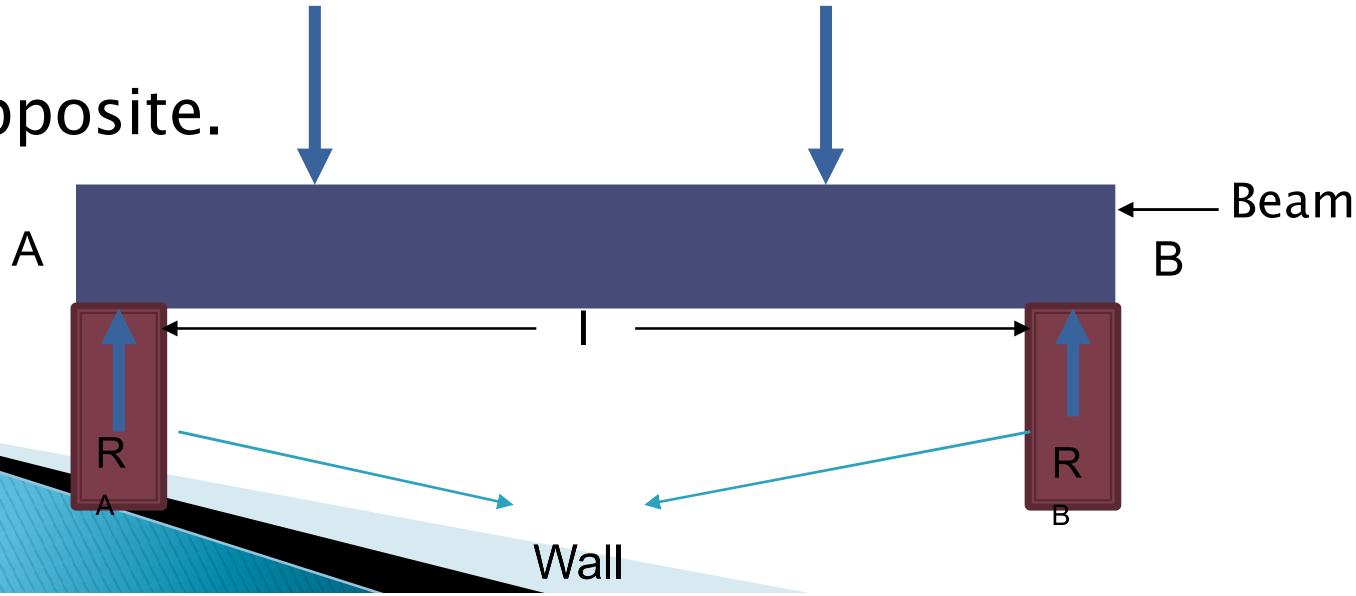
It is a beam which has a fixed support at one end and a simple support at the other end.



Support Reaction in Beams

Every Engineering Structure needs to be supported, so that, it can remain in equilibrium under any system of forces (called Action) likely to act on it. These supports develop the forces as Support Reaction.

Fig shows a beam AB carrying loads (action) supported on the walls, which tend to push the walls down. Since the walls are rigid, will resist the downward push. The walls will exert upward forces called Reactions. Actions are equal and opposite.



Support Reaction

- When a number of forces are acting on a beam (called action) then the support of beam will provide the reactions called Support Reaction (R_A and R_B).
- R_A and R_B along with loads will keep the system in equilibrium. ($\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma M = 0$)

Recalling of Earlier Concepts

1. In a coplanar non-concurrent force system, three conditions of equilibrium can be applied $\Sigma F_x = 0, \quad \Sigma F_y = 0 \quad \text{and} \quad \Sigma M = 0$

2. Draw the free body diagram of the given beam by showing all the forces and reactions acting on the beam.

3. Apply the three conditions of equilibrium to calculate the unknown reactions at the supports.

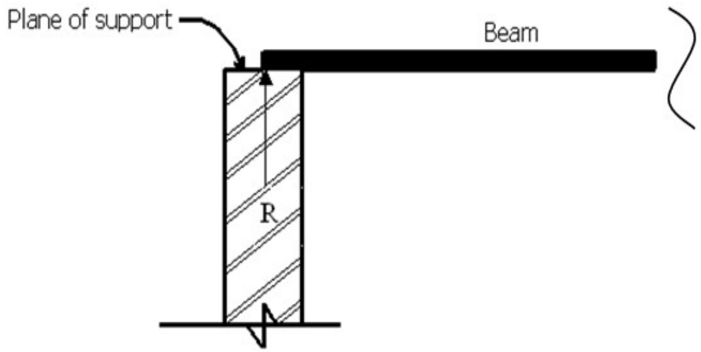
Supports

Supports are structures which prevent the beam or the body from moving and help to maintain equilibrium.

Types of Supports

1) Simple support

This is a support where a beam rests freely on a support. The beam is free to move only horizontally and also can rotate about the support. In such a support one reaction, which is perpendicular to the plane of support, is developed.



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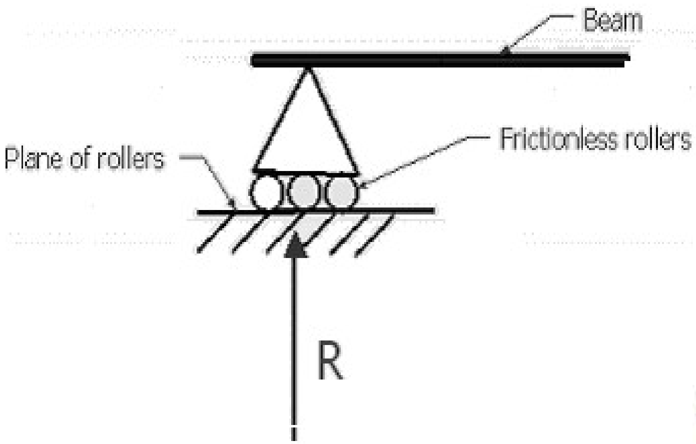

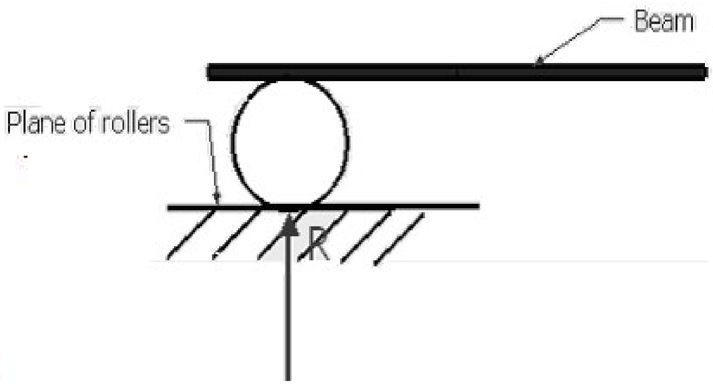

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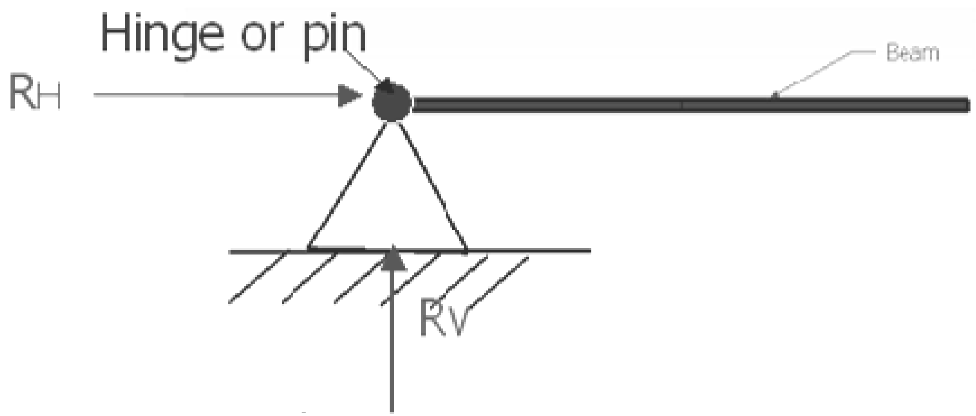

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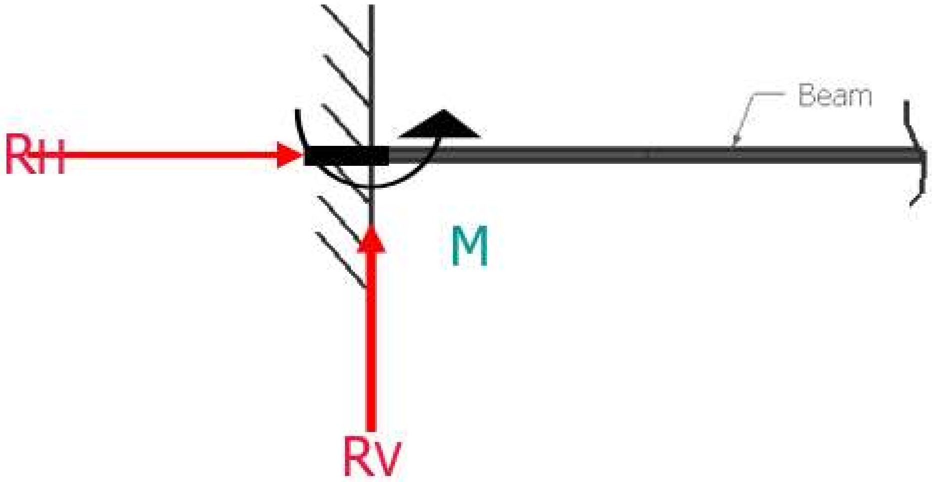
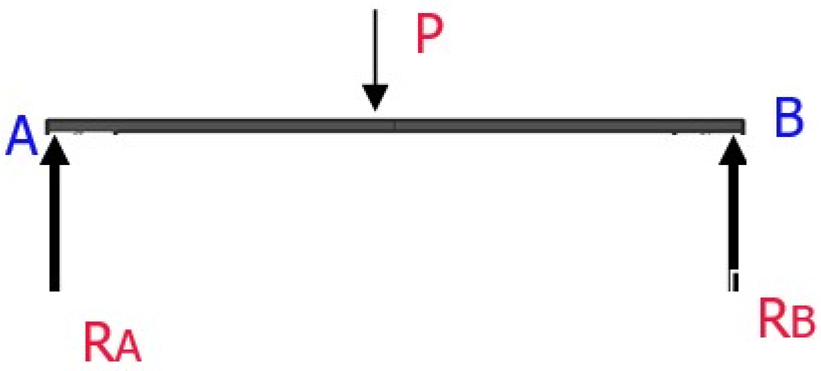
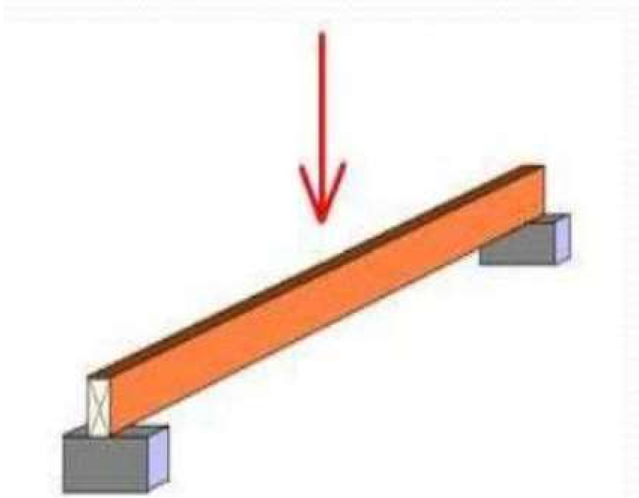


Types of loads

1) Point load or Concentrated load

If a load acts over a very small length of the beam, it is assumed to act at the mid-point of the loaded length and such a loading is termed as Point load or Concentrated load.

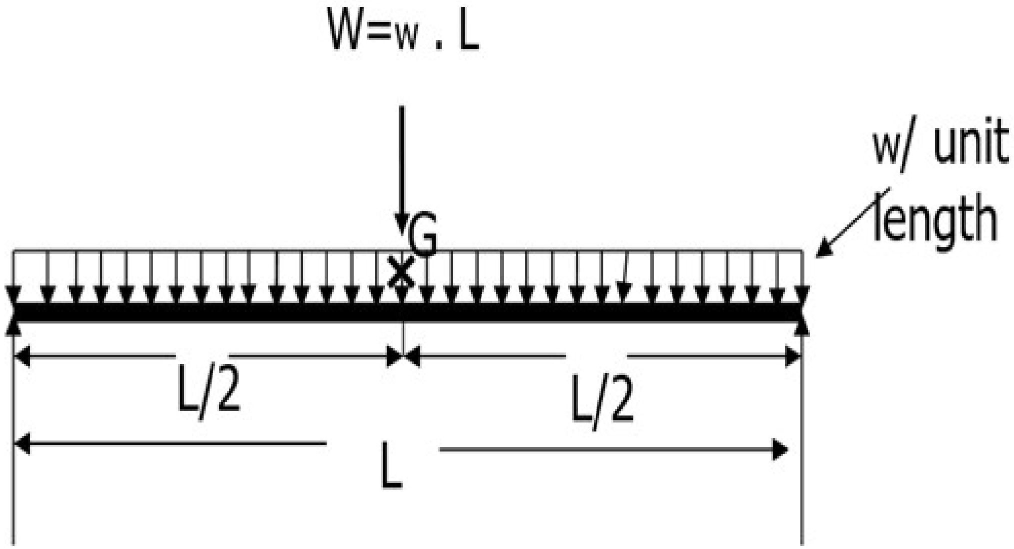


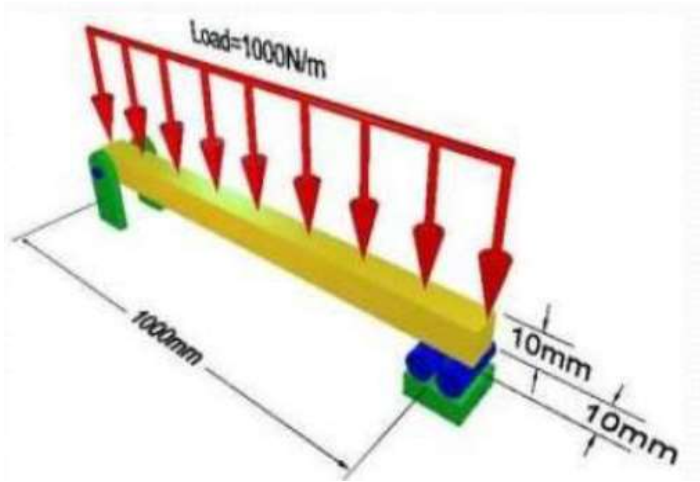


2) <u>Uniformly distributed load (UDL)</u>

If a beam is loaded in such a manner that each unit length of the beam carries the same intensity of loading, then such a loading is called UDL.

The UDL should be replaced by an equivalent point load or total load acting through the mid point of the loaded length.





3) <u>Uniformly varying load (UVL)</u>

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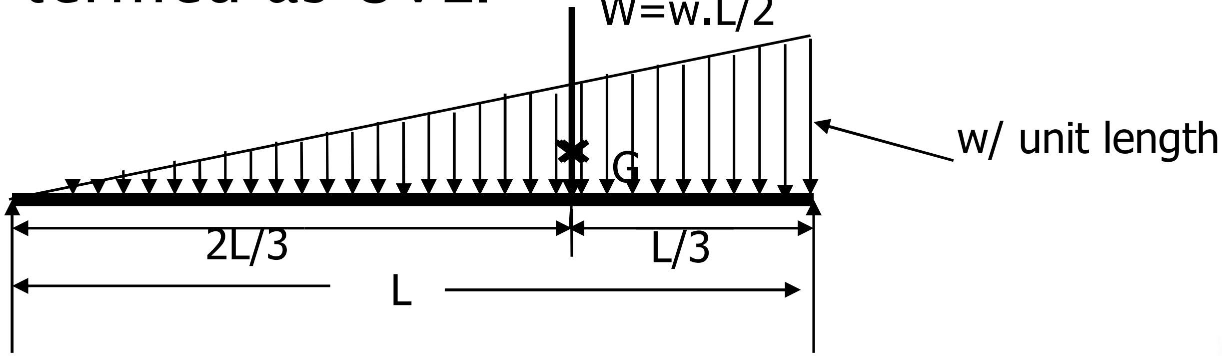
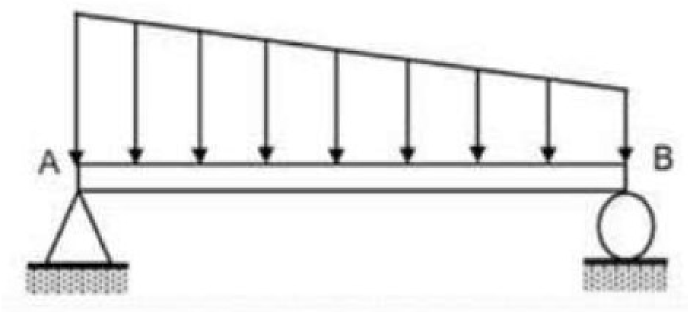
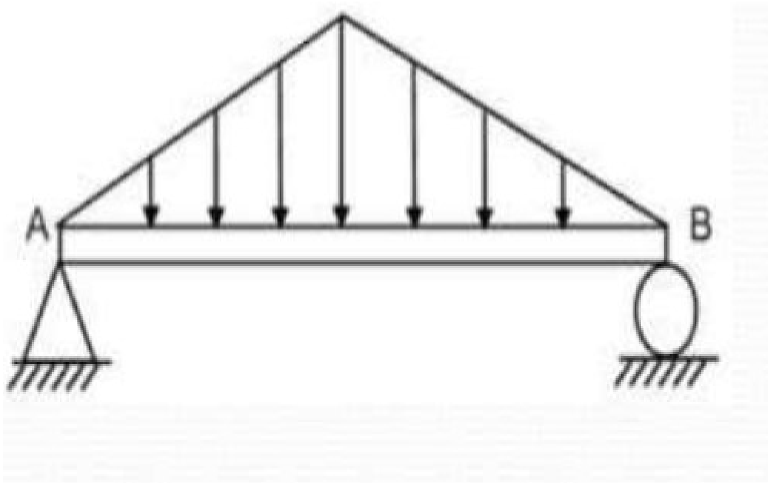
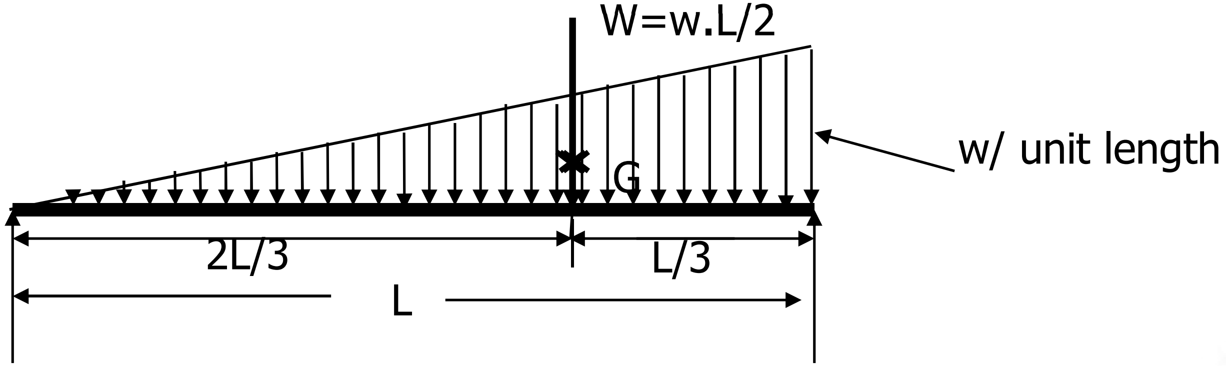


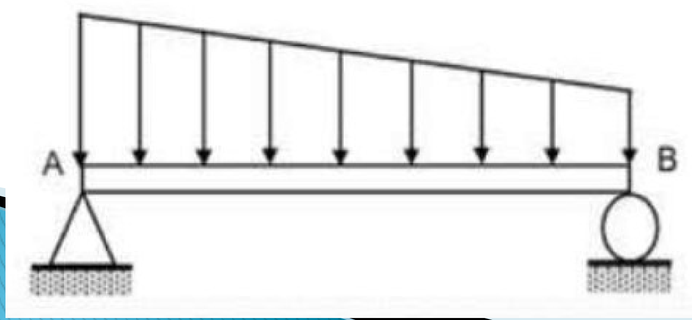


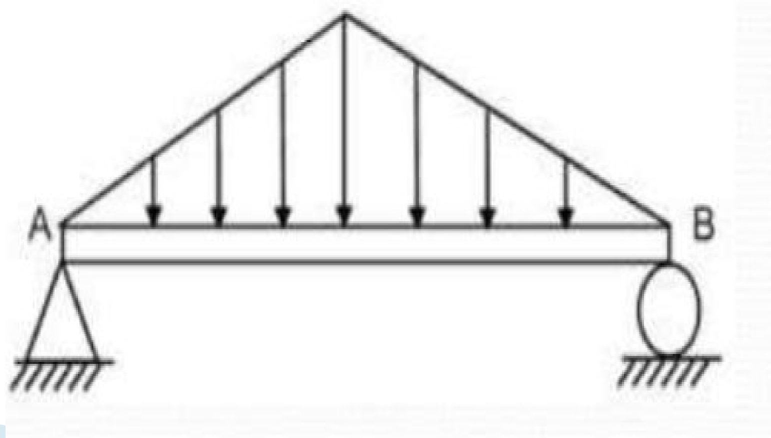


3) <u>Uniformly varying load (UVL)</u>

In applying conditions of equilibrium, a given UVL should be replaced by an equivalent point load or total load acting through the centroid of the loading diagram (right angle triangle). The magnitude of the equivalent point load or total load is equal to the area of the loading diagram.







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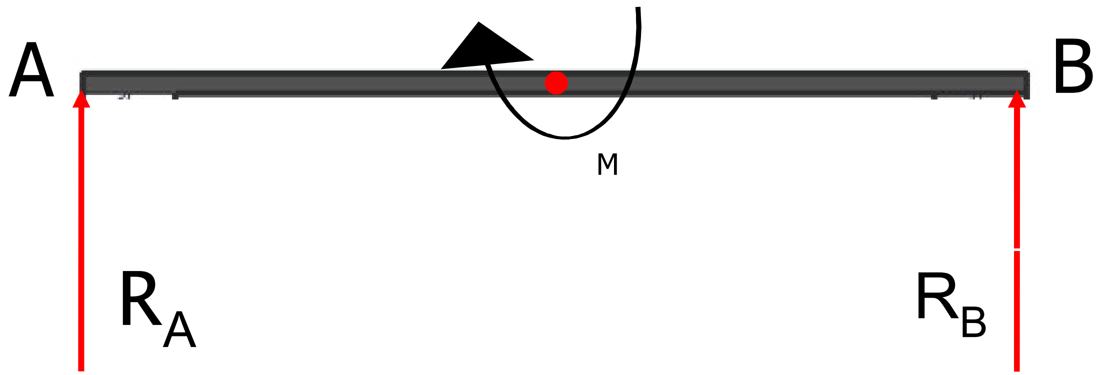

BEAM

- A beam is a structural member or element, which is in equilibrium under the action of a non-concurrent force system.

BEAM

- A beam is a structural member or element, which is in equilibrium under the action of a non-concurrent force system.
- The force system is developed due to the loads or forces acting on the beam and also due to the support reactions developed at the supports for the beam.
- In a beam, one dimension (length) is considerably larger than the other two dimensions (breath & depth). The smaller dimensions are usually neglected and as such a beam is represented as a line for theoretical purposes or for analysis.
- For the beam to be in equilibrium, the reactions developed at the supports the should be equal and opposite to the loads.

Types of beams

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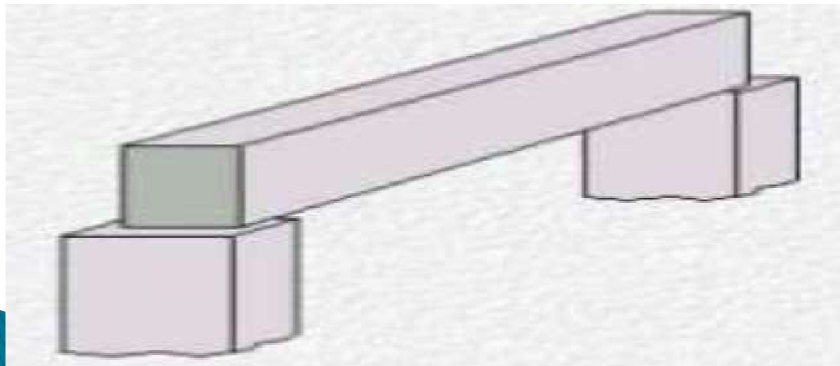
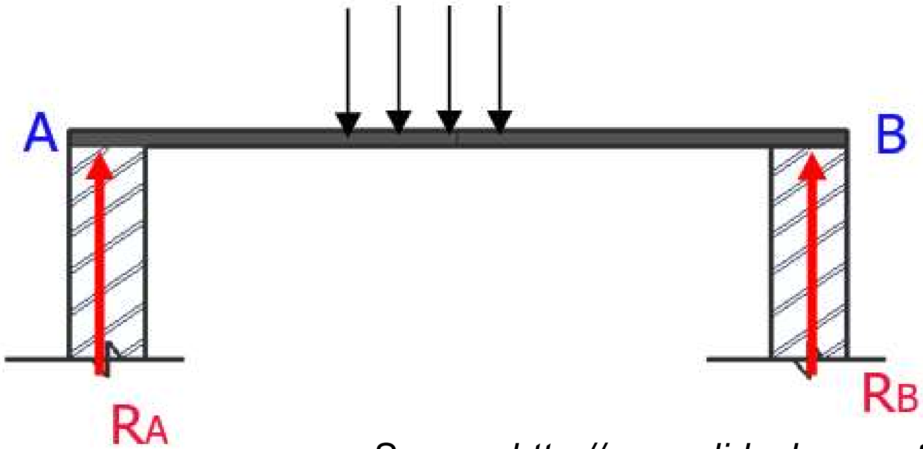
The equations of static equilibrium are sufficient to find all the unknown reactions. The types of beams which are statically determinate are as follows:

1) Simply Supported Beams

A beam is said to be simply supported when both ends of the beam rest on simple supports. Such a beam can carry or resist vertical loads only.

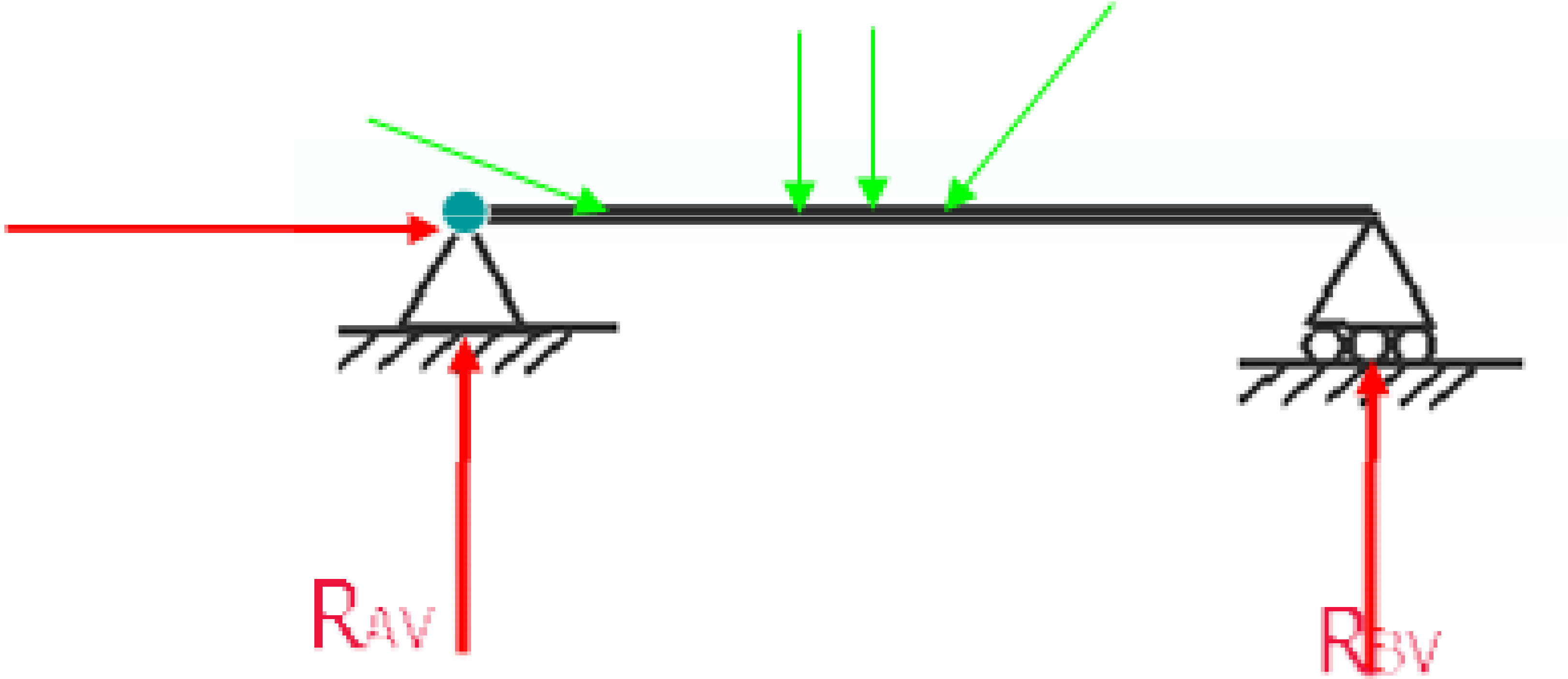




Source: http://www.slideshare.net

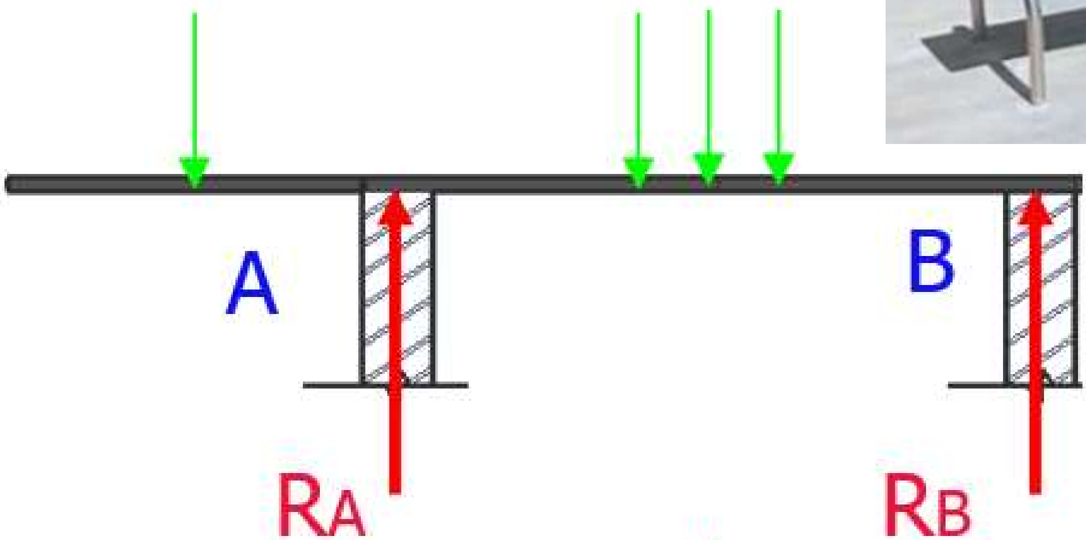
2) <u>Beam with one end hinged & other on rollers</u>

It is a beam where one end of the beam is hinged to a support and the other end rests on a roller support. Such a beam can carry any type of loads.



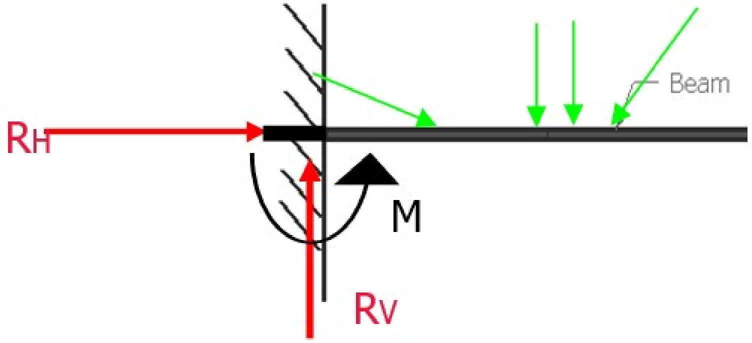
3) Over hanging beam

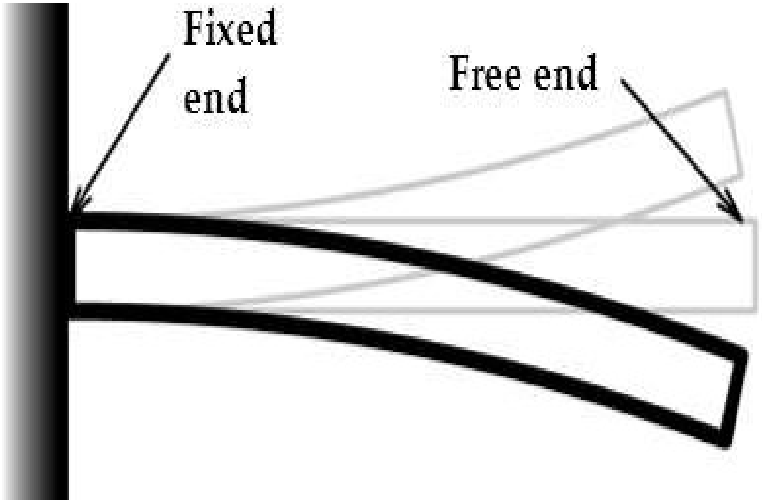
It is a beam which projects beyond the supports. A beam can have over hanging portions on one side or on both sides.



4) Cantilever Beams

It is a beam, with one end fixed and other and free. Such a beam can carry loads in any directions.



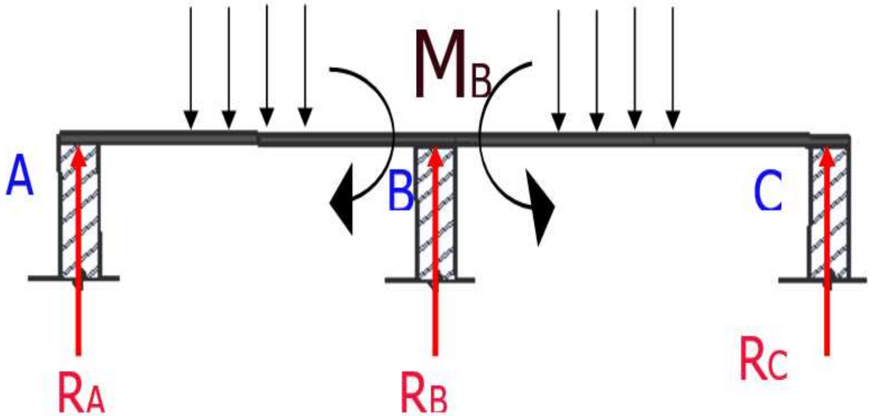


B) Statically Indeterminate Beams

In statically indeterminate beams, the number of unknown reactions exceeds the number of equations of static equilibrium and hence cannot be solved completely using these equations alone.

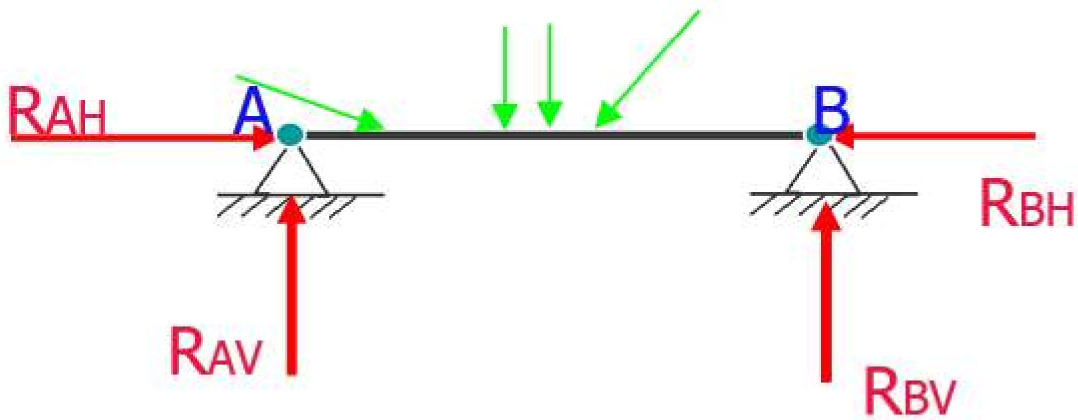
1) Continuous Beams

It is a beam which rests over a series of supports at more than two points.



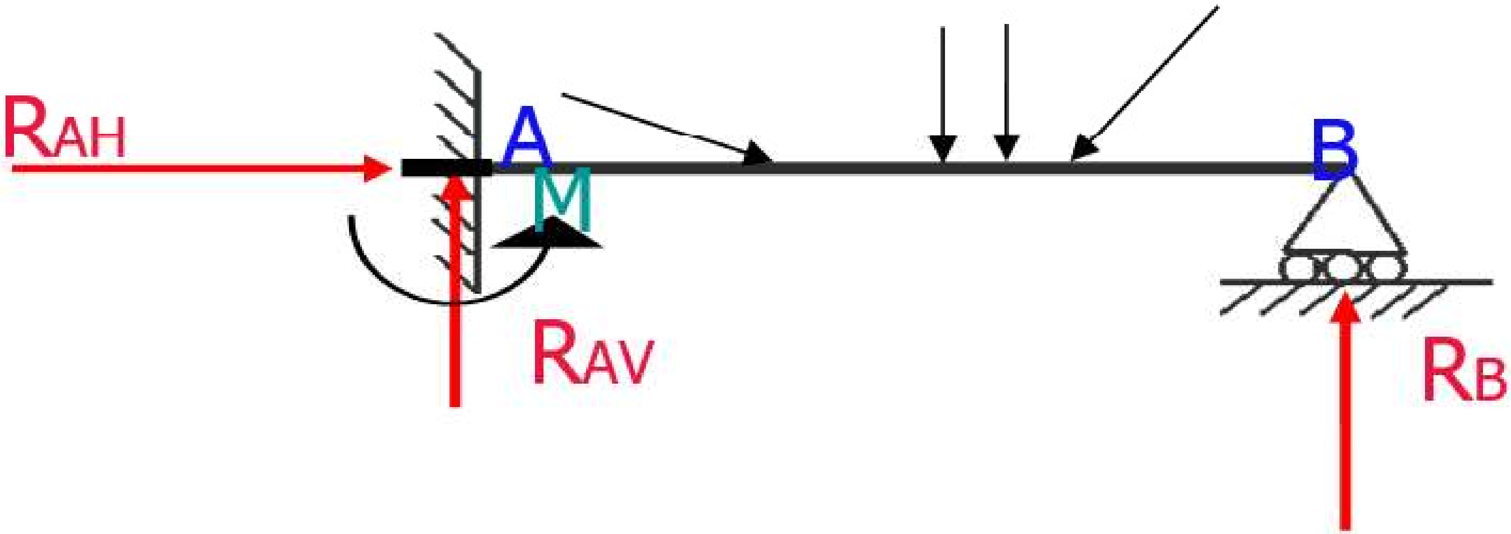
2) <u>Hinged Beam (Beams with both ends hinged)</u>

It is a beam which is hinged to supports at both ends. Such a beam can carry loads is any direction.



3) <u>Propped Cantilever Beam</u>

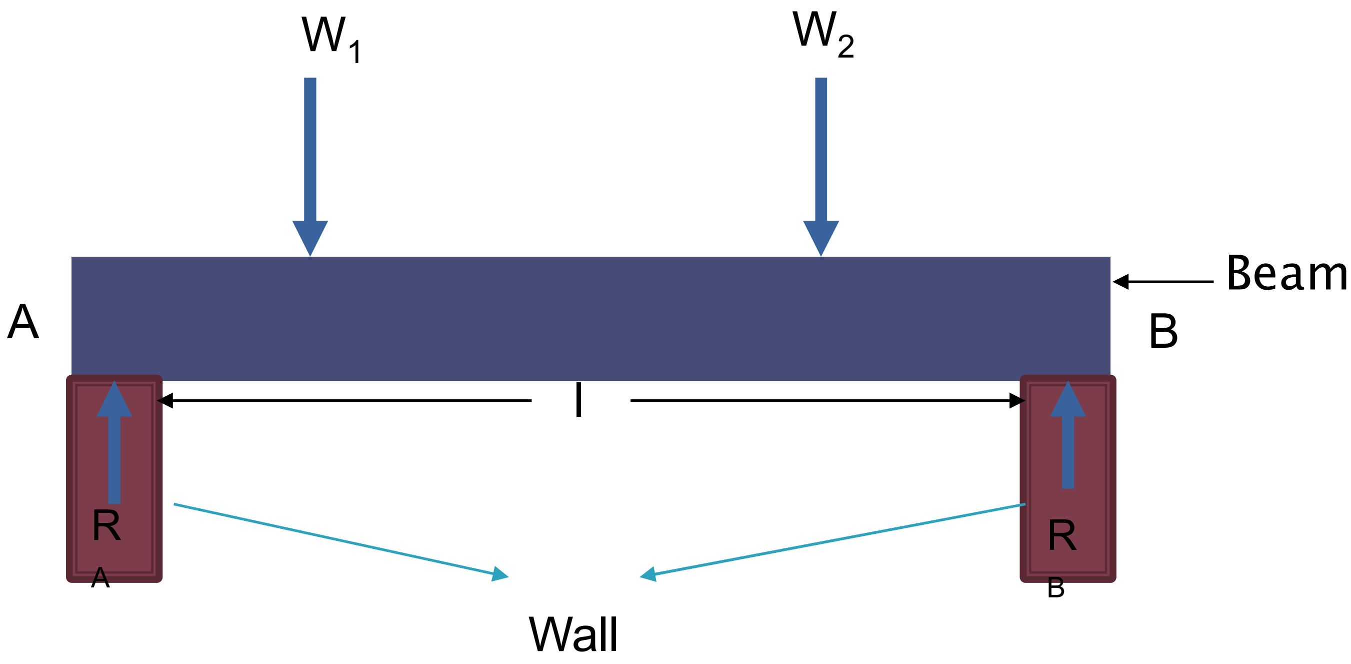
It is a beam which has a fixed support at one end and a simple support at the other end.



Support Reaction in Beams

Every Engineering Structure needs to be supported, so that, it can remain in equilibrium under any system of forces (called Action) likely to act on it. These supports develop the forces as Support Reaction.

Fig shows a beam AB carrying loads (action) supported on the walls, which tend to push the walls down. Since the walls are rigid, will resist the downward push. The walls will exert upward forces called Reactions. Actions are equal and opposite.



Support Reaction

- When a number of forces are acting on a beam (called action) then the support of beam will provide the reactions called Support Reaction (R_A and R_B).
- R_A and R_B along with loads will keep the system in equilibrium. ($\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma M = 0$)

Recalling of Earlier Concepts

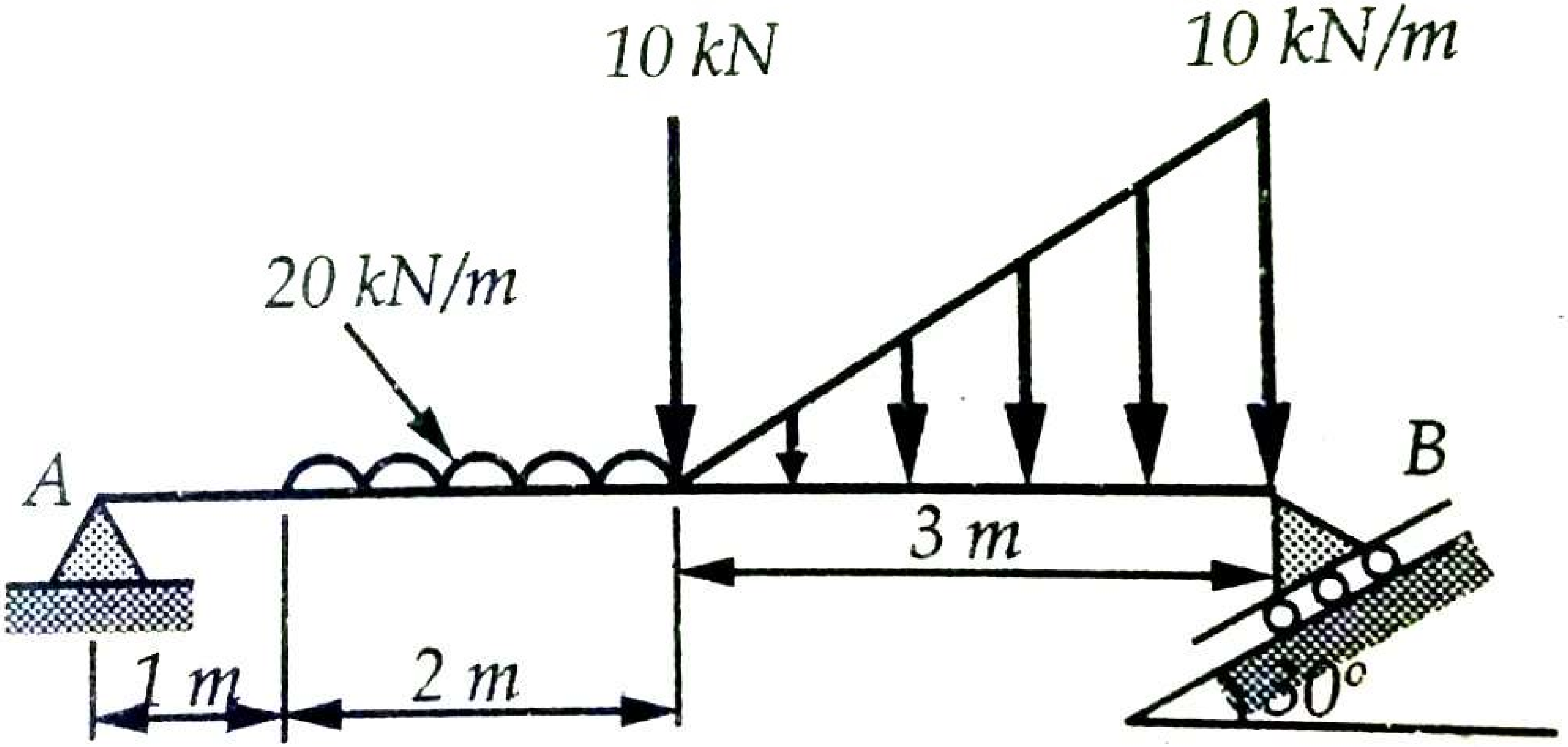
1. In a coplanar non-concurrent force system, three conditions of equilibrium can be applied, n $\Sigma F_x = 0, \quad \Sigma F_y = 0 \quad \text{and} \quad \Sigma M = 0$

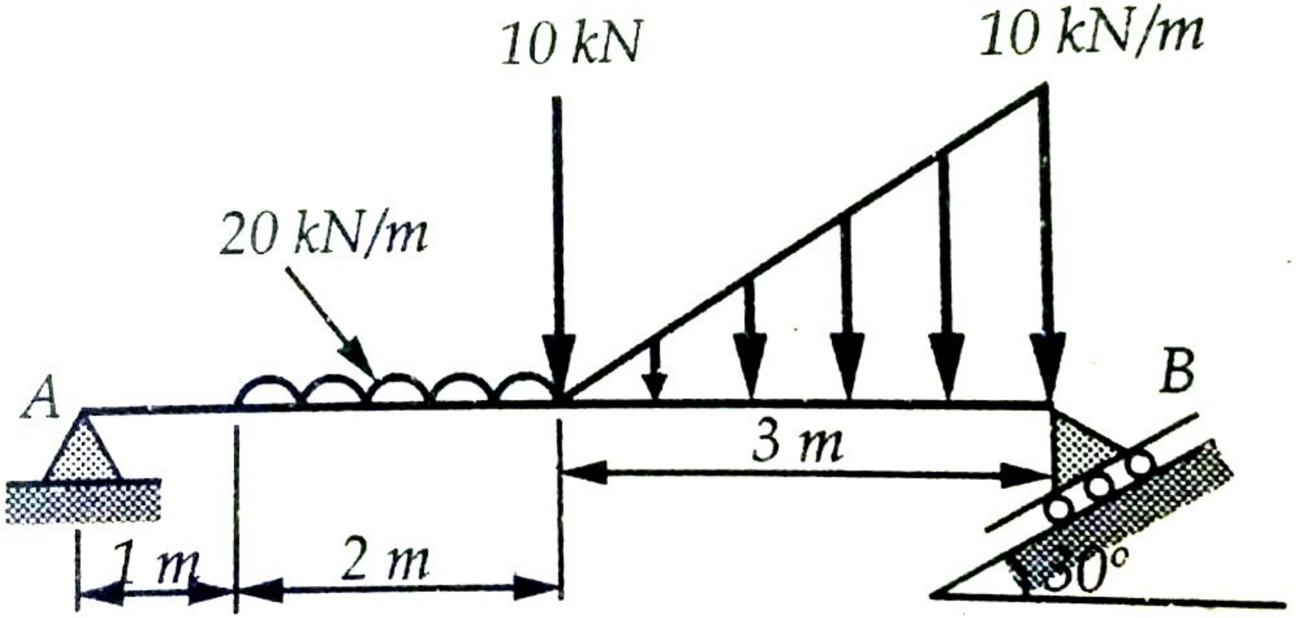
2. Draw the free body diagram of the given beam by showing all the forces and reactions

acting on the beam.

3. Apply the three conditions of equilibrium to calculate the unknown reactions at the supports.

2) Determine the reactions at the supports for the beam loaded shown in fig.





$\Sigma M_A = 0$

$+(40*2) + (10*3) + (15*5) - (R_B \text{ Sin } 60 *6) = 0$

$R_B = 35.6 \text{ kN}$

$\Sigma F_x = 0$

$A_x - R_B \cos 60 = 0$

$A_x = 17.8 \text{ kN}$

$F_y = 0$

$A_y - 40 - 10 - 15 + R_B \text{ Sin } 60 = 0$

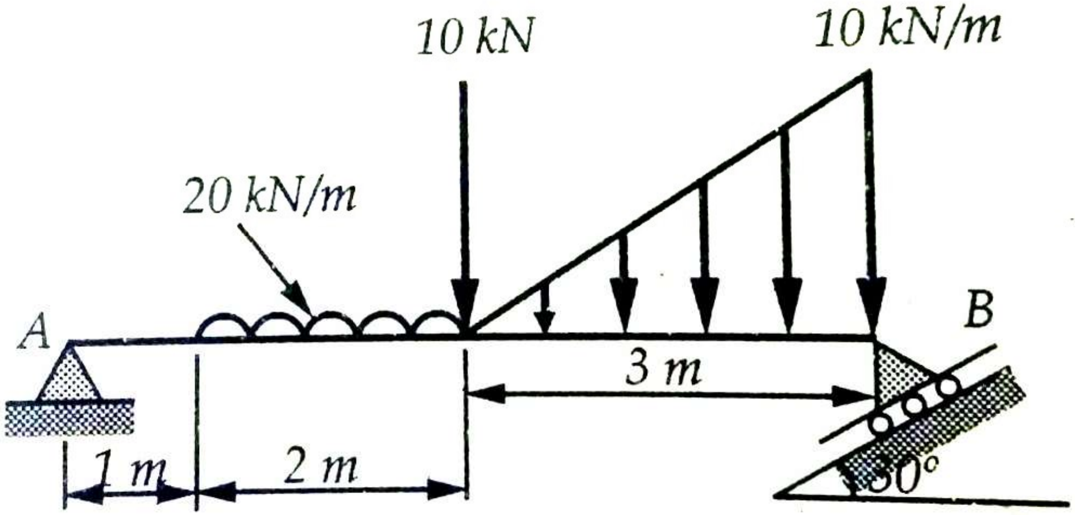
$A_y = 34.17 \text{ kN}$

$R_A = \text{sq rt } (A_x^2 + A_y$

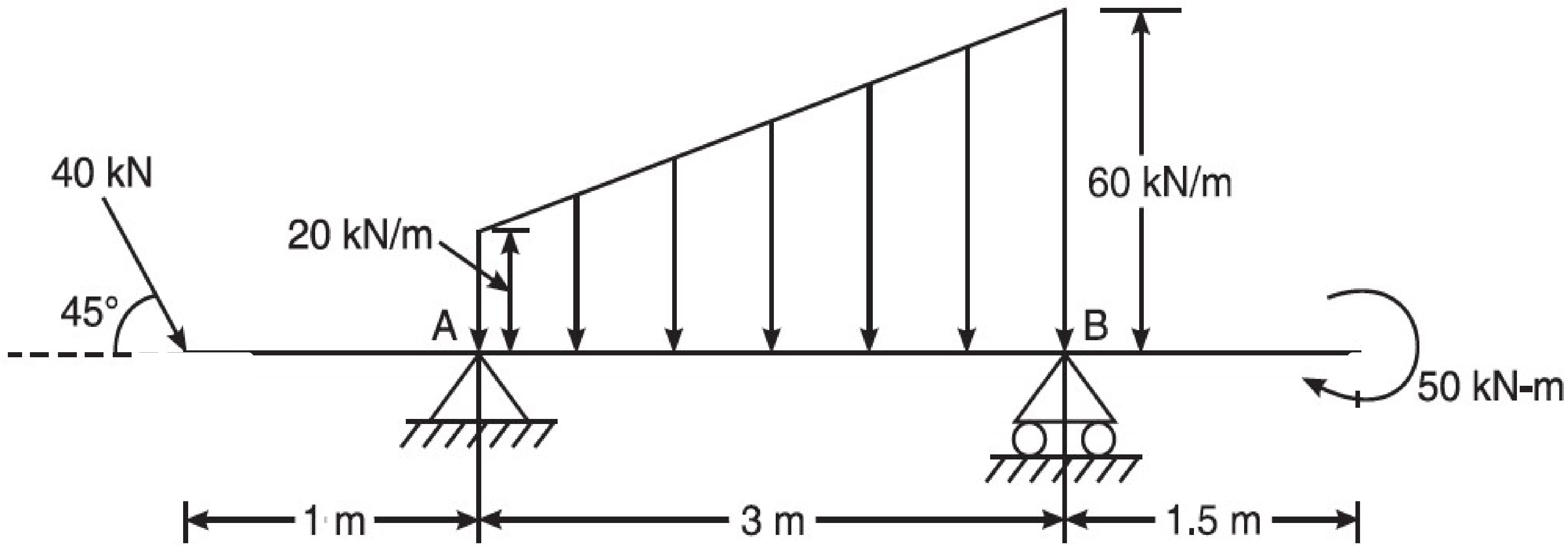
$R_A = 38.53 \text{ kN}$

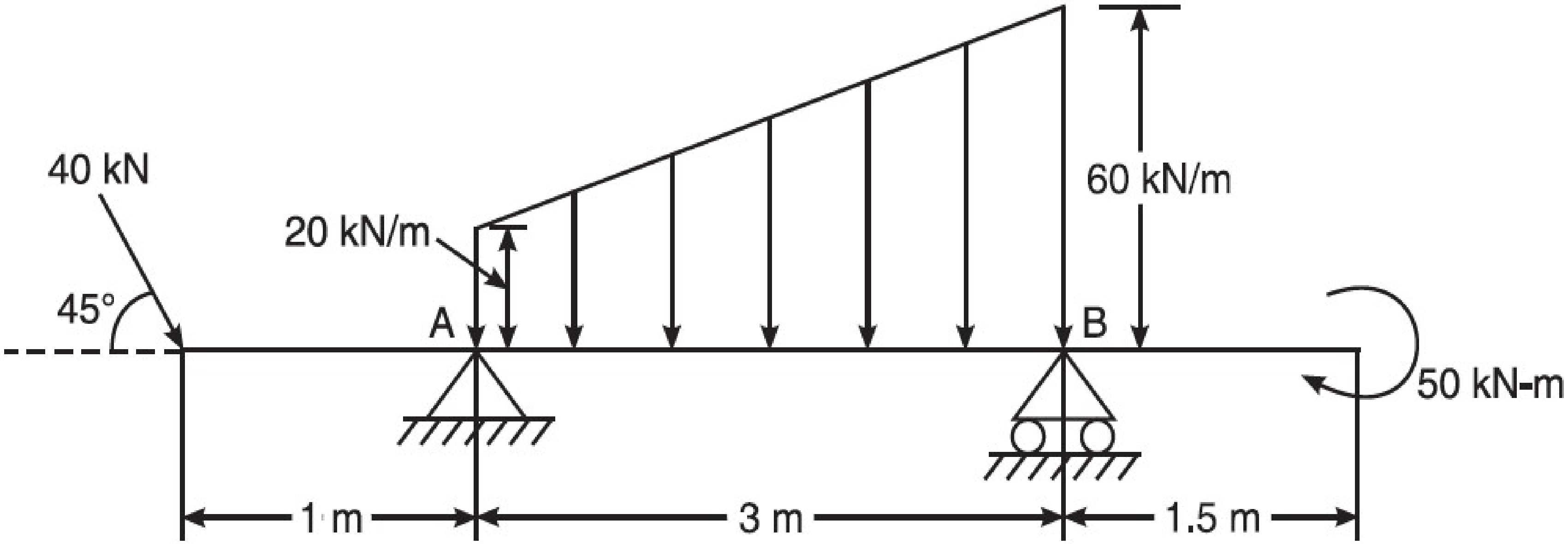
$\theta_A = \tan^{-1} (A_y \,/\, A_x)$

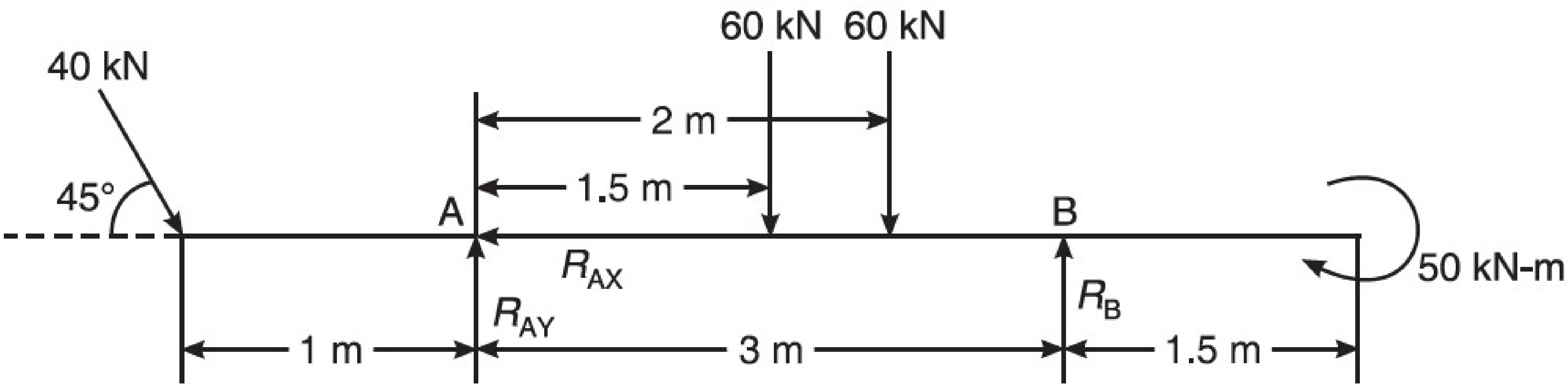
$\theta_A = 62.48^o$

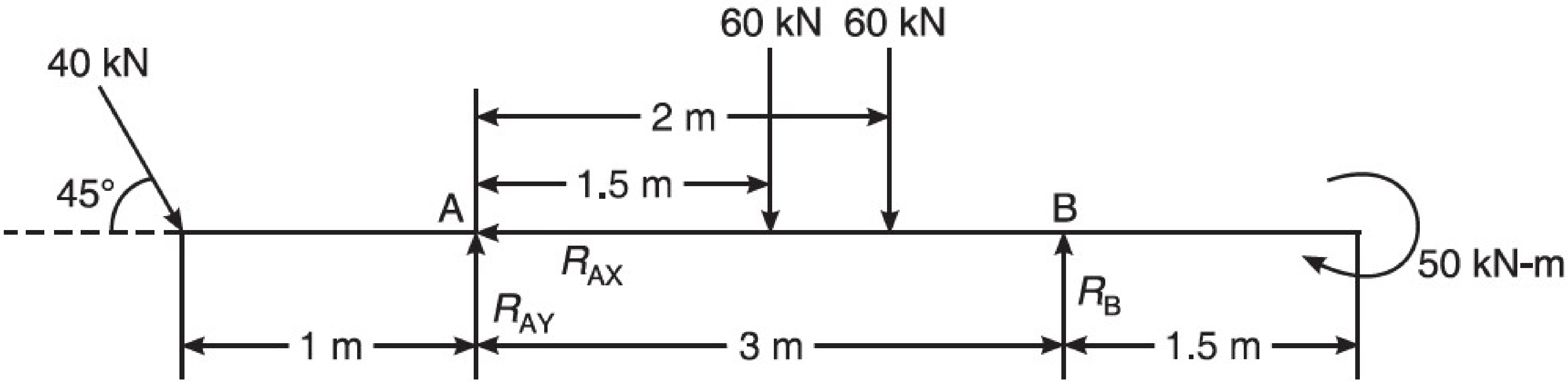


3) Determine the reactions at the supports A and B for a beam loaded as shown in fig.




40 kN
45°
20 kN/m
A
B
60 kN/m
50 kN-m
1 m
3 m
1.5 m


40 kN
45°
60 kN
60 kN
2 m
1.5 m
A
B
R_{AX}
R_{AY}
R_B
50 kN-m
1 m
3 m
1.5 m



Using the conditions of equilibrium

$\Sigma F_x = 0$

$40 \cos 45° - R_{AX} = 0$

$R_{AX} = 28.284$ kN

$\Sigma F_y = 0$

$R_{AY} + R_B - 40 \sin 45° - 60 - 60 = 0$

$R_{AY} + R_B = 148.284$ .... (i)

$\Sigma M_A = 0$

$-R_B \; *3 + 60 * 1.5 + 60 \; *2 + 50 - 40 \sin 45° *$
$1 = 0$

$R_B = 77.238$ kN

Substituting the value of R_B in (i), we get

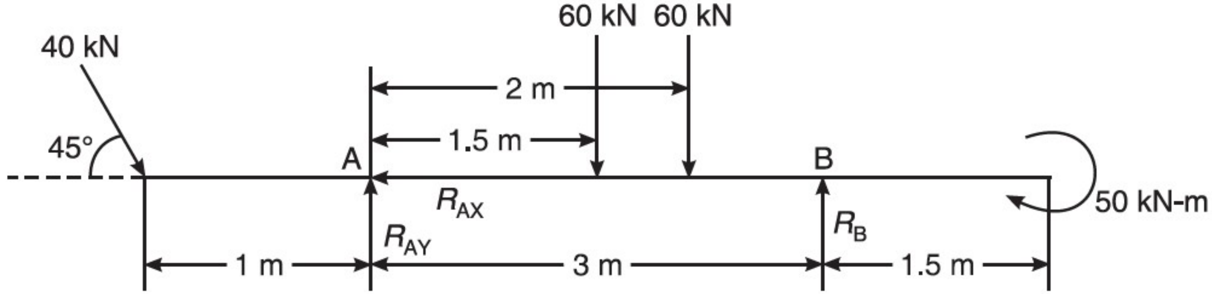
$R_{AY} = 71.046$ kN

$R_A = \text{sq rt} (A_x^2 + A_y^2)$

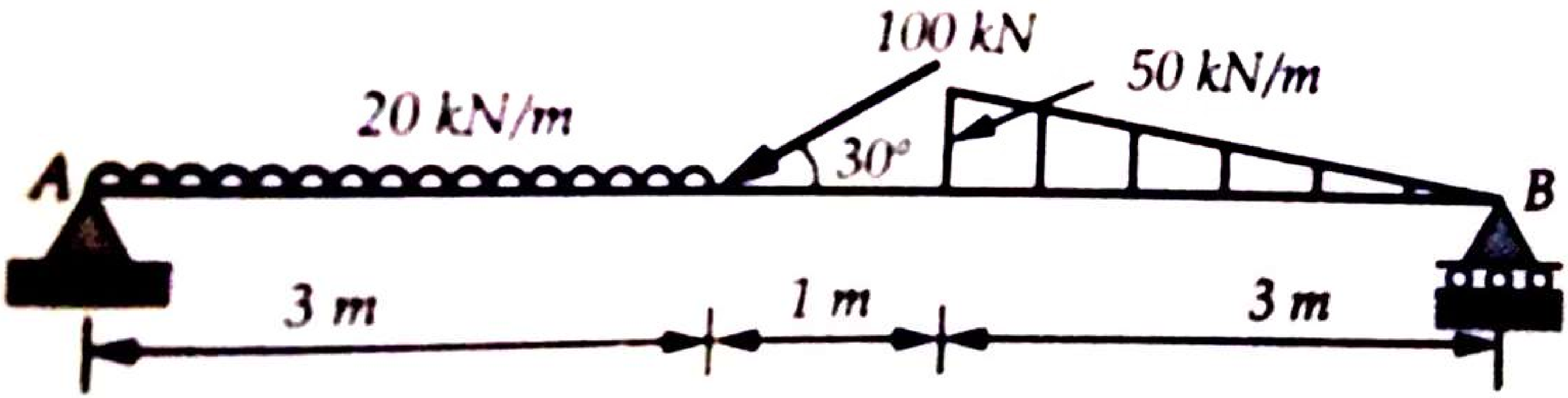
$R_A = 78.469$ kN

$\theta_A = \tan^{-1} (A_y / A \;)$

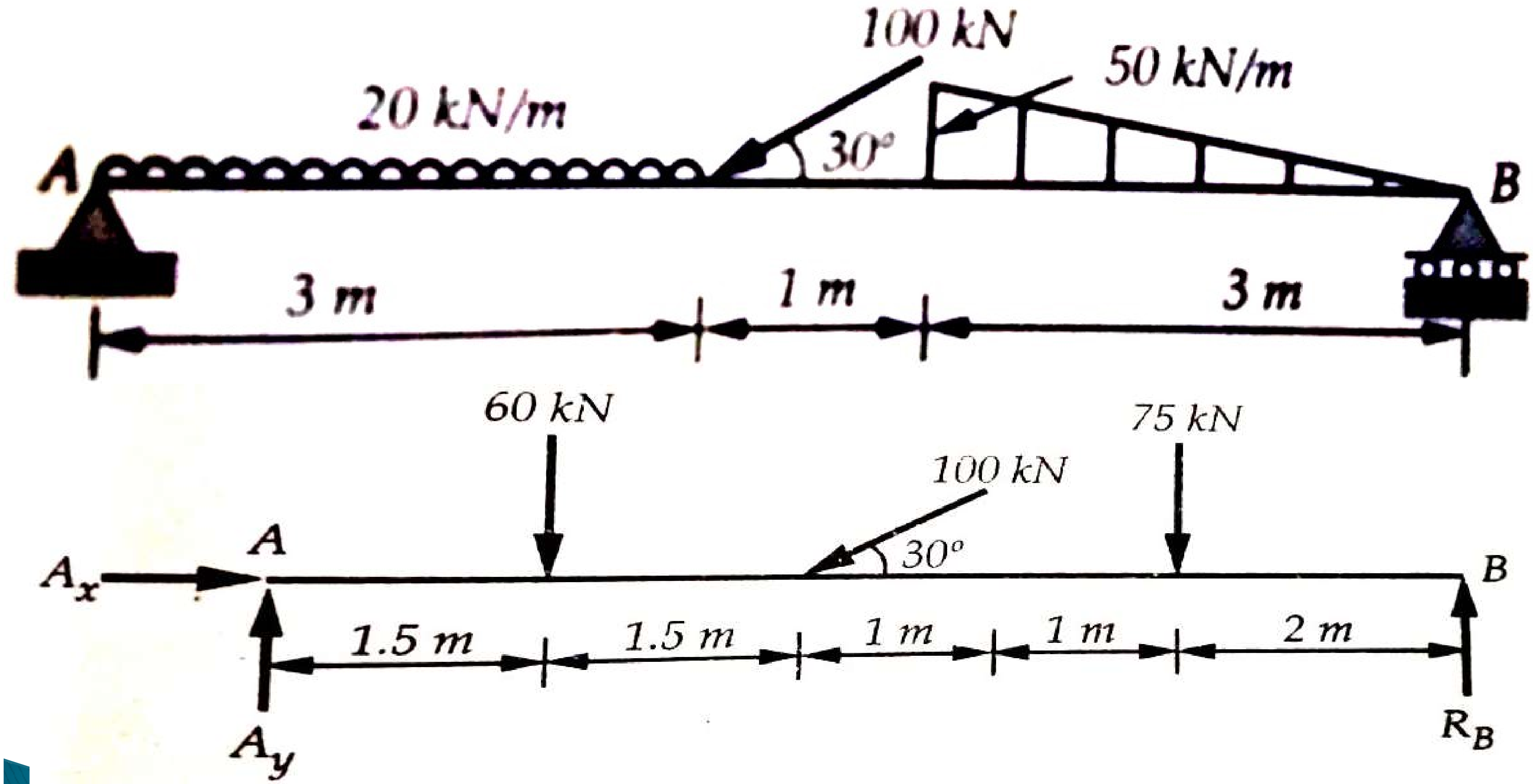
$\theta_A = 68.29°$

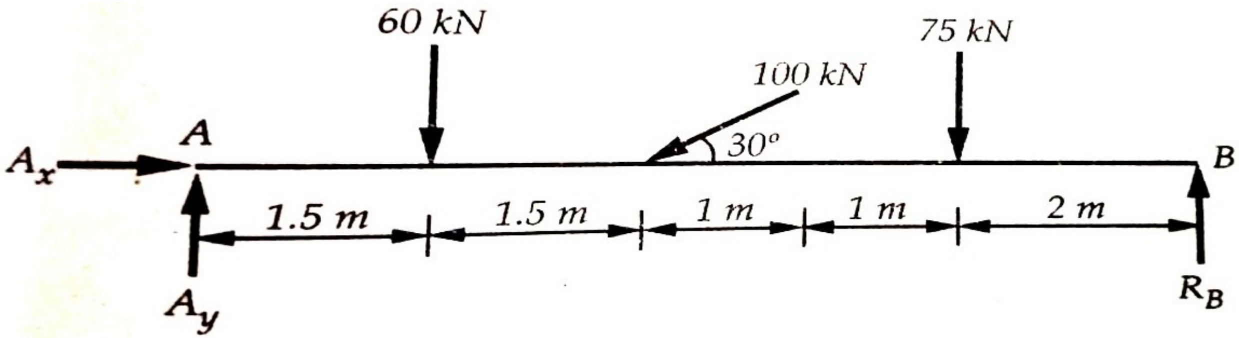


4) Determine the reactions at the support for the beam shown in fig.



4) Determine the reactions at the support for the beam shown in fig.





$\Sigma M_A = 0$

$(7 * R_B) - (60*1.5) - (100 \sin 30 *3) - (75*5) = 0$

$R_B = 87.86$ kN

$\Sigma F_x = 0$

$A_X - 100 \cos 30 = 0$

$A_X = 86.6$ kN

$\Sigma F_y = 0$

$A_Y - 60 - 100 \sin 30 - 75 + R_B = 0$

$A_Y = 97.14$ kN

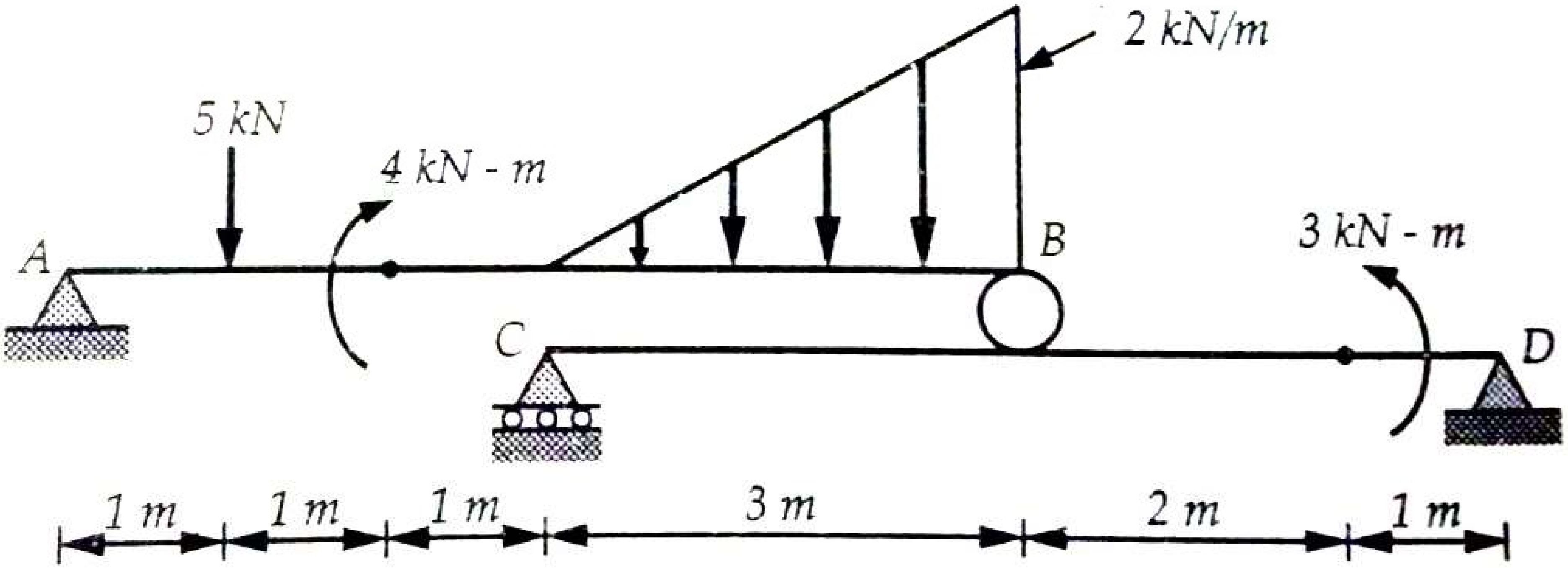
$R_A = \text{sq rt} (A_x^2 + A_y^2)$

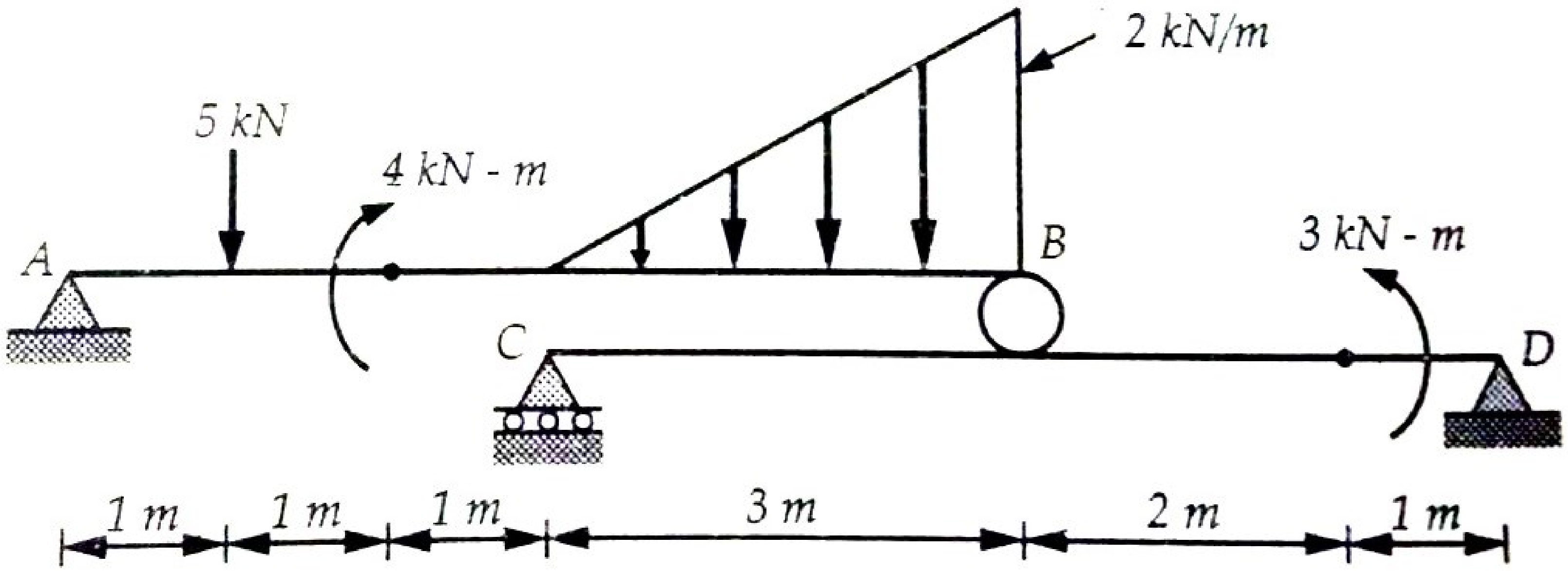
$R_A = 130.14$ kN

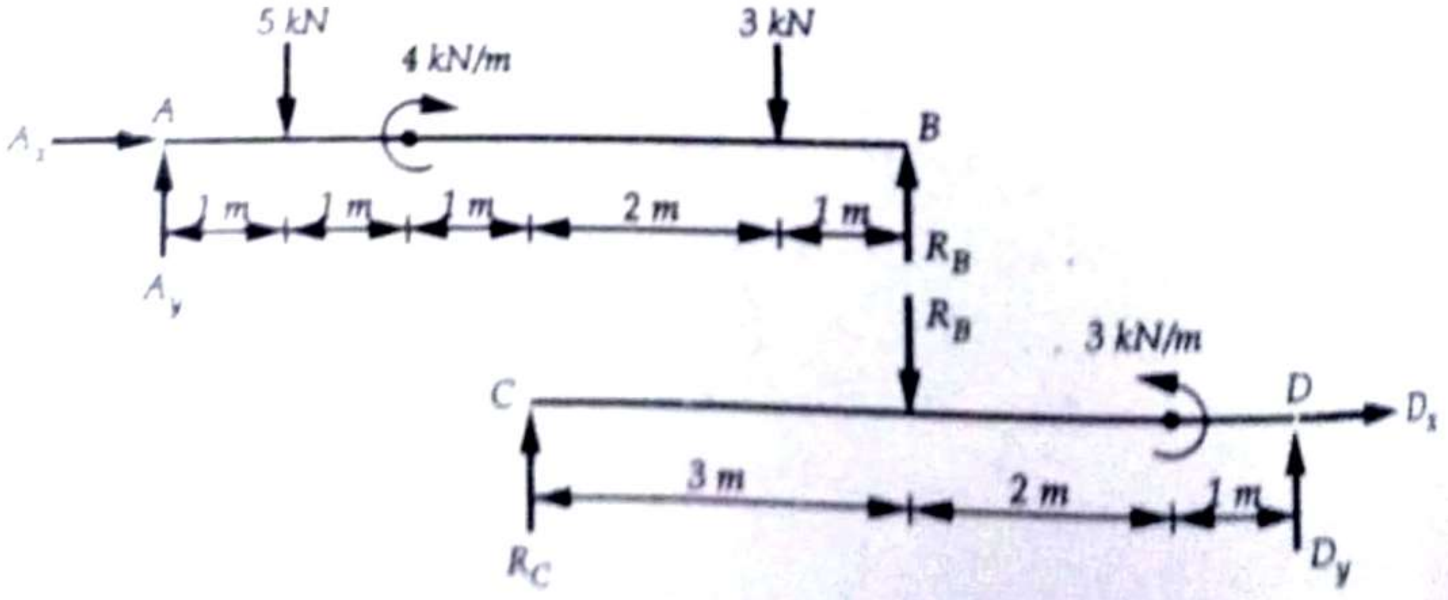
$\theta_A = \tan^{-1} (A_y / A_x)$

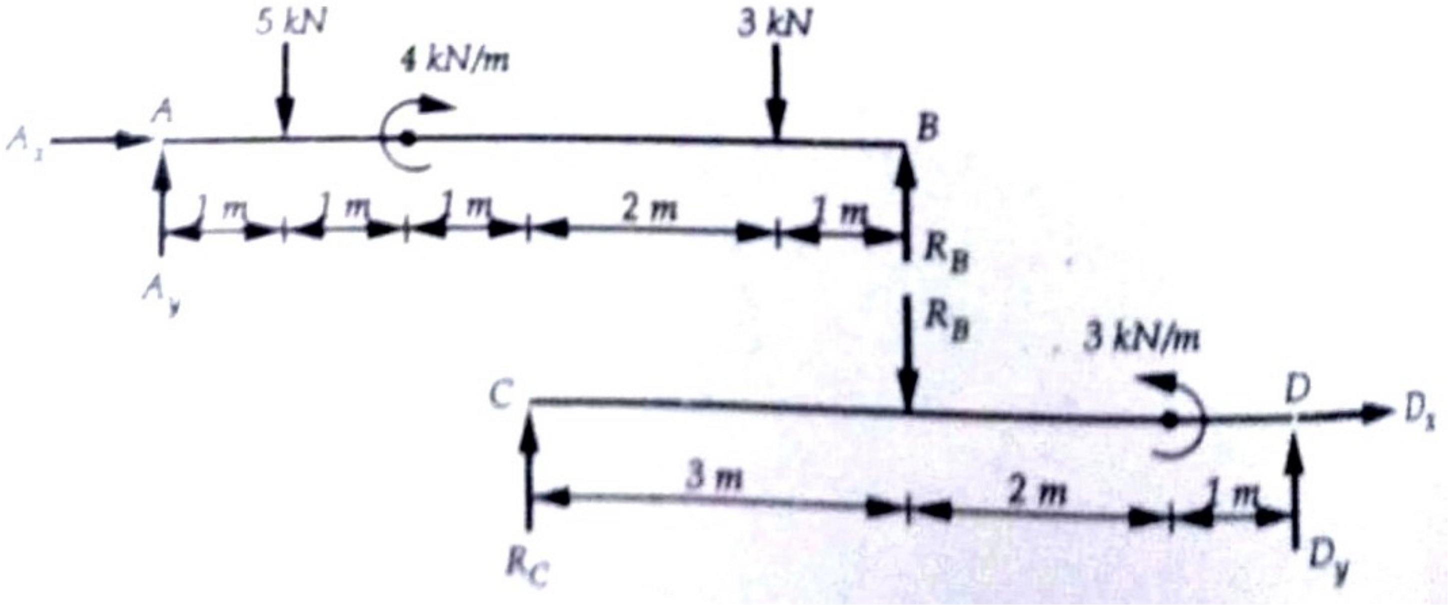
$\theta_A = 48.28^o$

5) Determine the reactions at the ends of the beams AB and CD as shown in fig. Neglect the self weight of the beams.




2 kN/m
5 kN
4 kN - m
A
B
3 kN - m
C
D
1 m
1 m
1 m
3 m
2 m
1 m


5 kN
3 kN
4 kN/m
A
B
1 m
1 m
1 m
2 m
1 m
R_B
A_y
R_B
3 kN/m
C
D
D_x
3 m
2 m
1 m
R_C
D_y



<u>For Beam AB</u>

$\Sigma F_x = 0$

$A_x = 0$

$\Sigma M_A = 0$

$(5*1) + 4 + (3*5) - (R_B*6) = 0$

$R_B = 4$ kN

$\Sigma F_y = 0$

A_Y-5-3 $R_B = 0$

$A_Y = 4$ kN

For Beam CD

$\Sigma F_x = 0$

$D_x = 0$

$\Sigma M_C = 0$

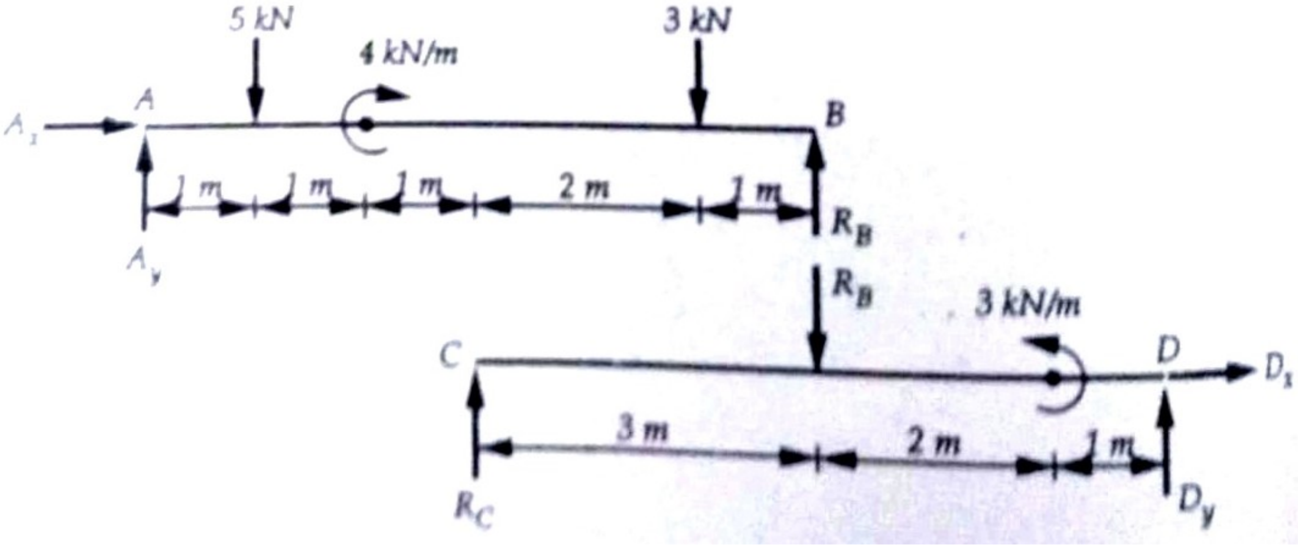
$-(R_B * 3) + 3 -$

$D_Y = 1.5 \text{ kN}$

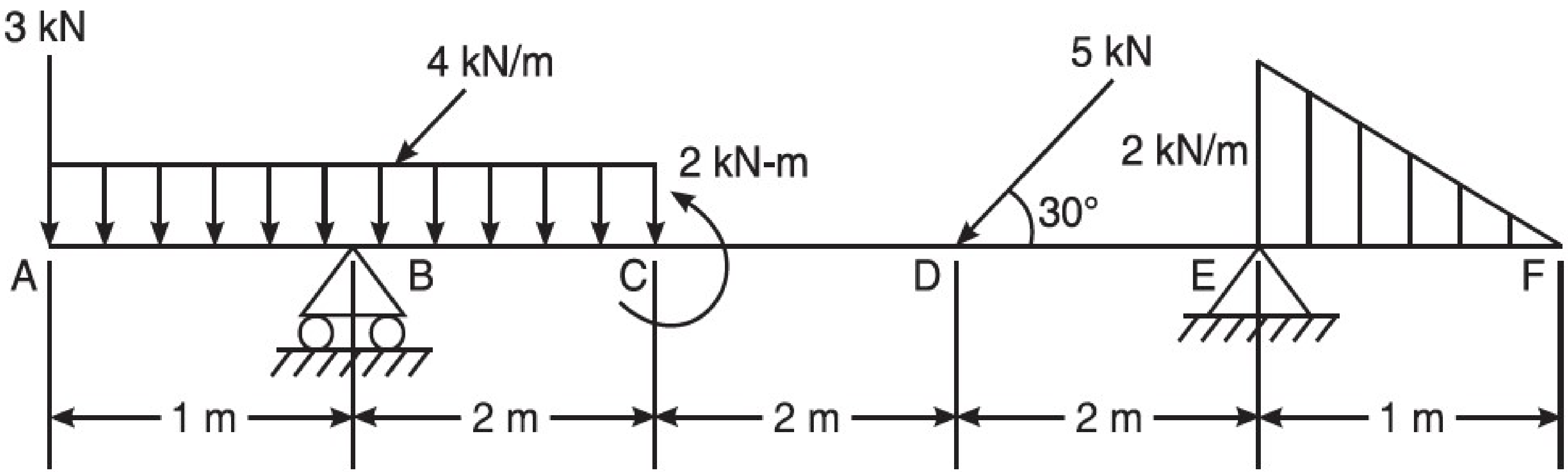
$\Sigma F_y = 0$

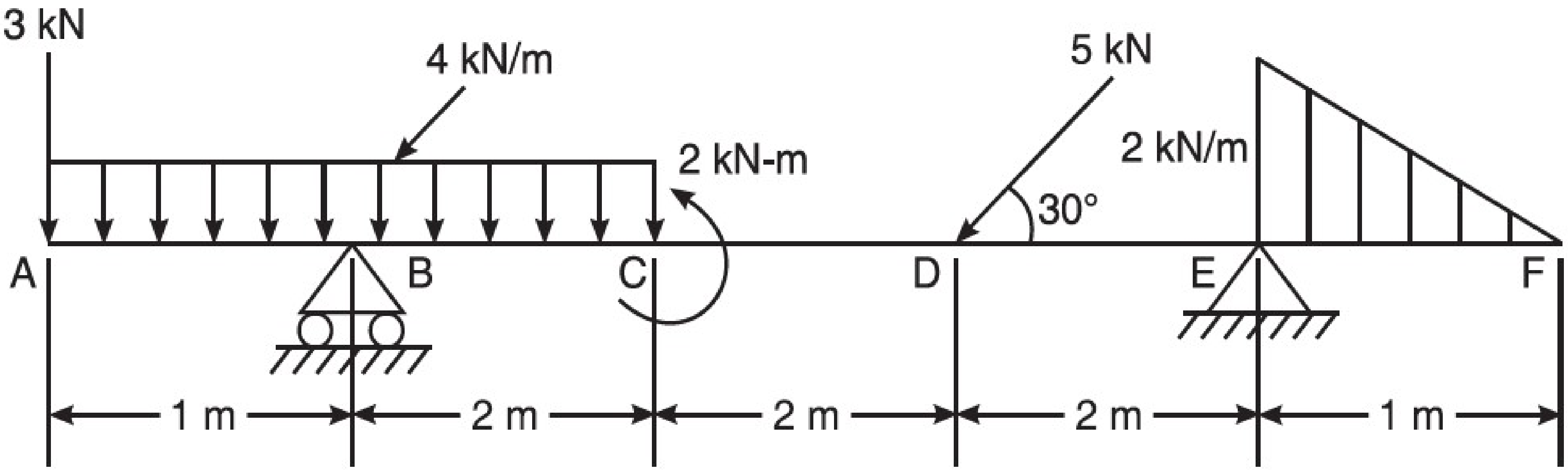
$D_Y - R_B + R_C = 0$

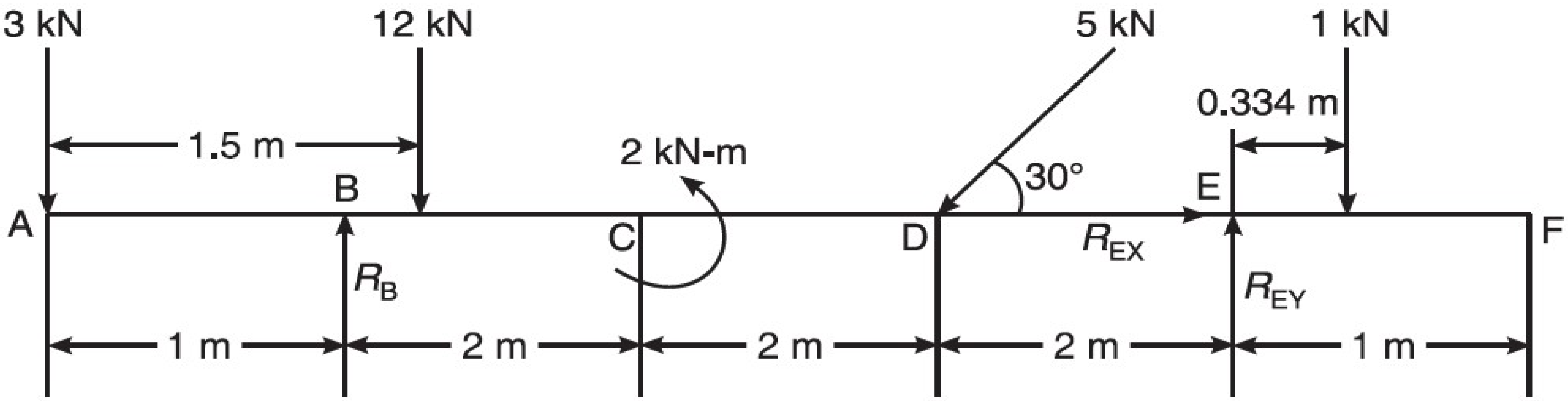
$R_C = 2.5 \text{ kN}$



6) Find the support reactions of the beam loaded as shown in Figure




3 kN
4 kN/m
2 kN-m
5 kN
30°
2 kN/m
A
B
C
D
E
F
1 m
2 m
2 m
2 m
1 m


3 kN
12 kN
5 kN
1 kN
0.334 m
1.5 m
2 kN-m
30°
A
B
C
D
E
F
R_{EX}
R_B
R_{EY}
1 m
2 m
2 m
2 m
1 m

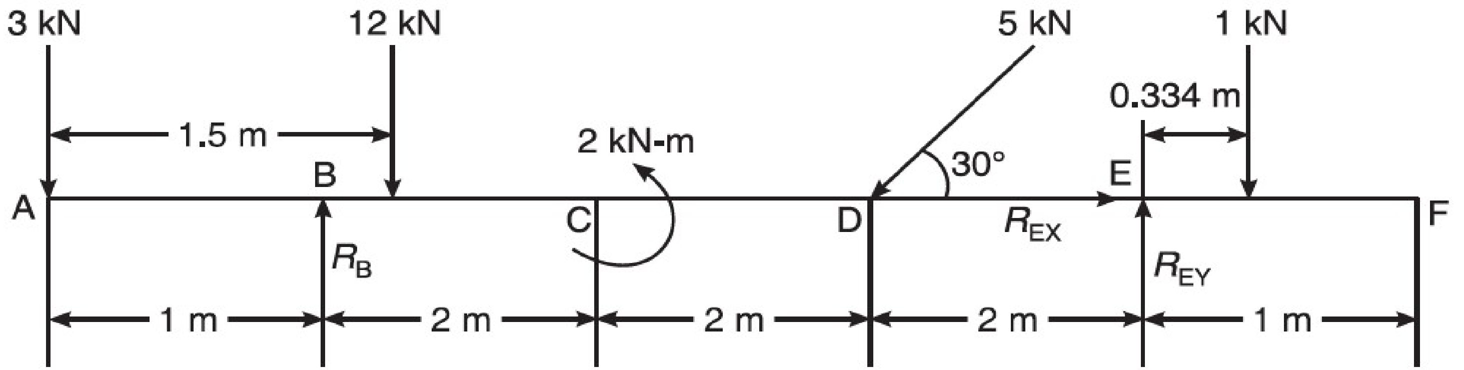
$$\Sigma F_x = 0$$

$$R_{EX} - 5 \cos 30° = 0 \qquad R_{EX} = 4.33 \text{ kN}$$

$$\Sigma F_Y = 0$$

$$R_B + R_{EY} - 3 - 12 - 5 \sin 30° - 1 = 0$$

$$R_B + R_{EY} = 18.5 \text{ ..... (i)}$$



$\Sigma M_B = 0$

$-R_{EY}$ *6 + 12 * 0.5 – 3* 1 – 2 + 5 sin 30° * 4 + 1*
6.334 = 0

$R_{EY} = 2.889$ kN

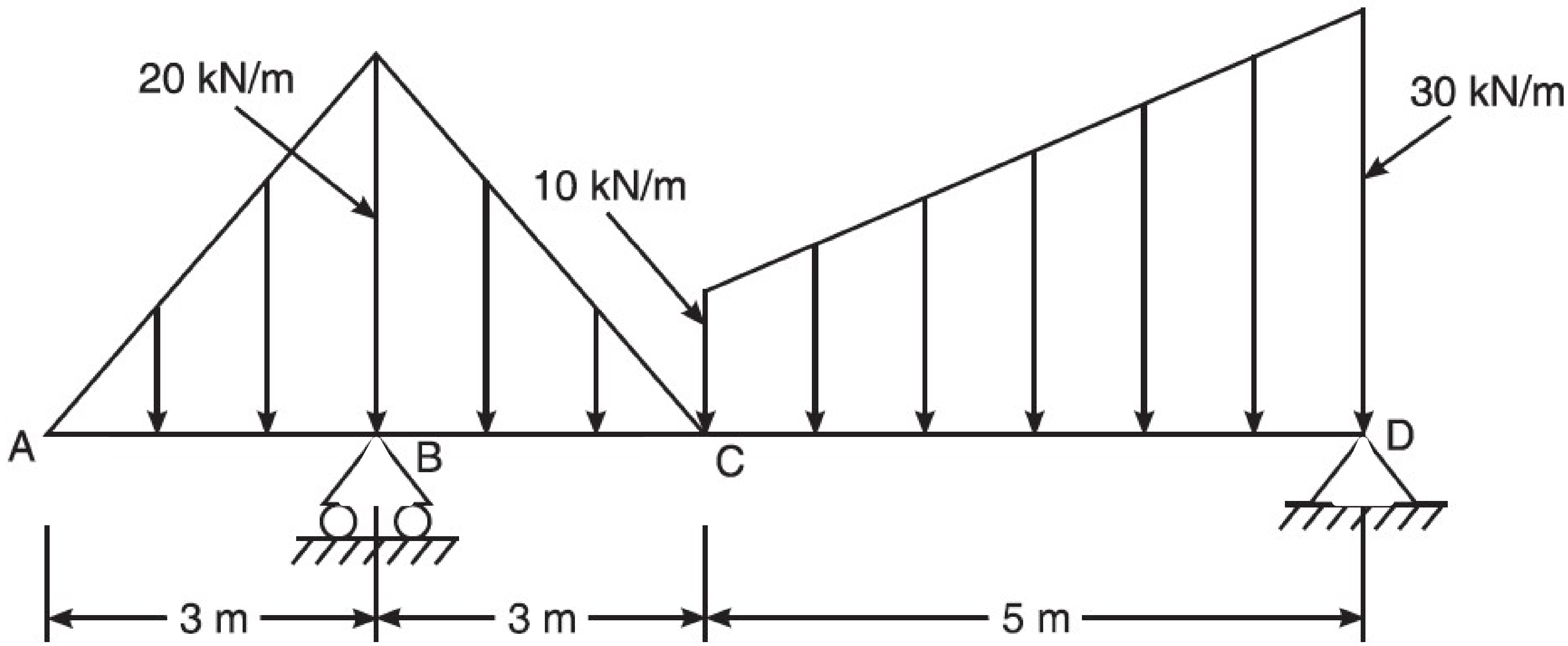
Substituting the value of R_{EY} in (i)

$R_B = 15.611$ kN

R_E = sq rt $(E_x^2 + E_y^2)$ $R_E = 5.2$ kN

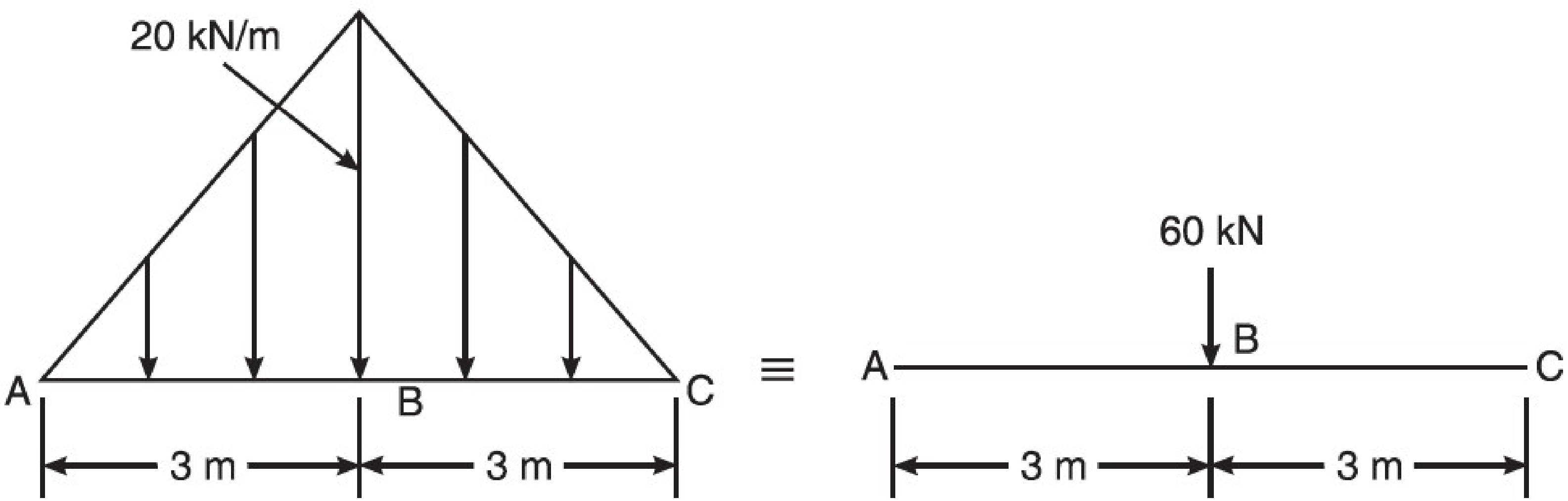
$\theta_E = \tan^{-1}(E_y / E_x)$ $\theta_E = 33.71°$

7) Determine the support reactions for the beam shown in Figure

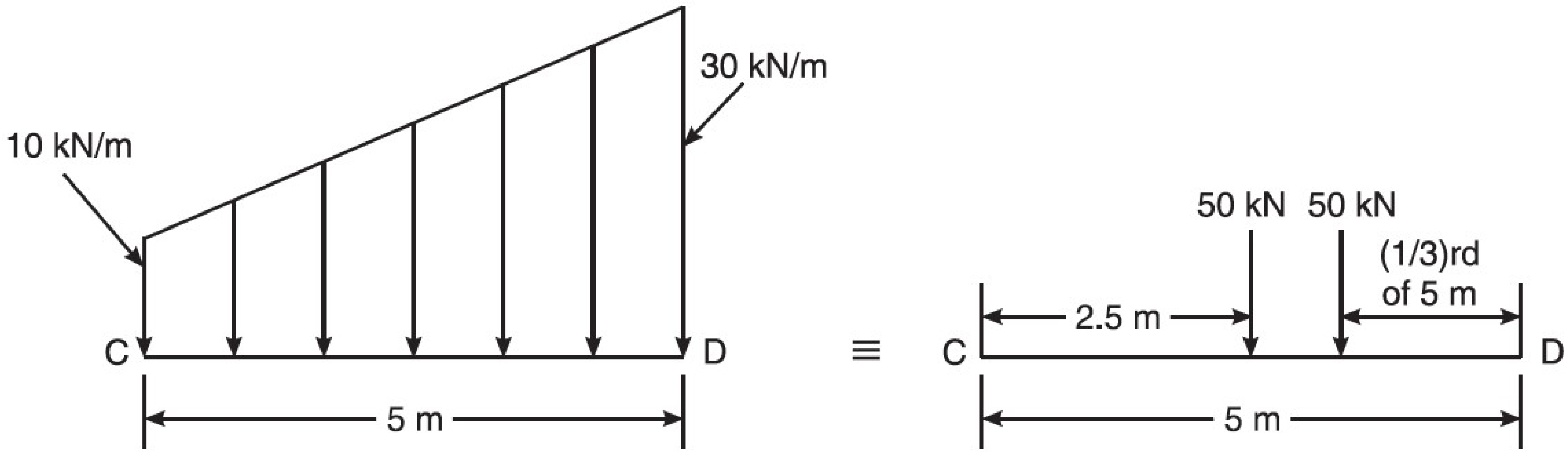


Triangular load into a point load

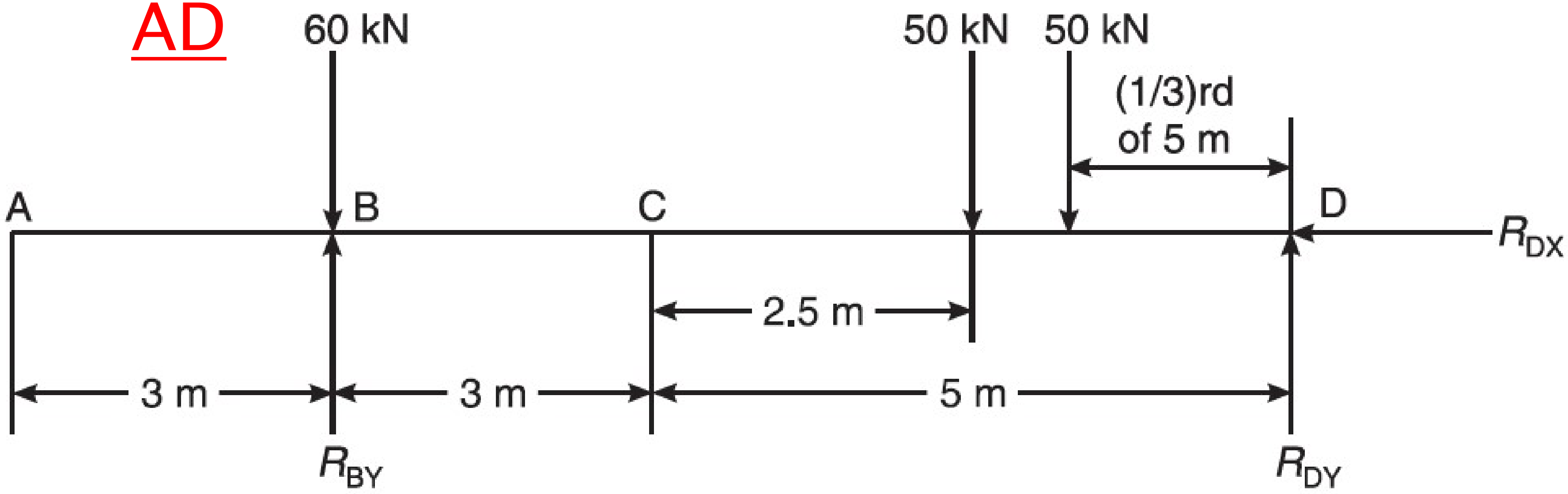
Area of the triangle =0.5 * 6*20 =60 kN



uniformly varying load to point load



FBD of Beam AD



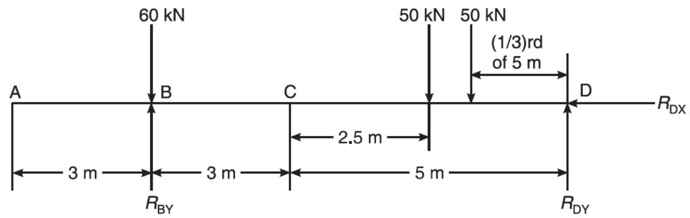
Using the conditions of equilibrium

$\Sigma F_x = 0$

$-R_{DX} = 0$

$\Sigma F_Y = 0$

$R_{BY} + R_{DY} - 60 - 50 - 50 = 0$



$R_{BY} + R_{DY} = 160$

$\Sigma M_B = 0$

$-(R_{DY} * 8) + \{50 *[(2/3)*5+3]\} + \{50[2.5+3\}) = 0$

$R_{DY} = 73.958$ kN

$R_{BY} = 160 - 73.958 = 86.042$ kN

$R_D = \text{sq rt } (D_x^2 + D_y^2)$ $R_D = 73.958$ kN

8) Find the support reactions at A and B for the beam loaded as shown in Figure

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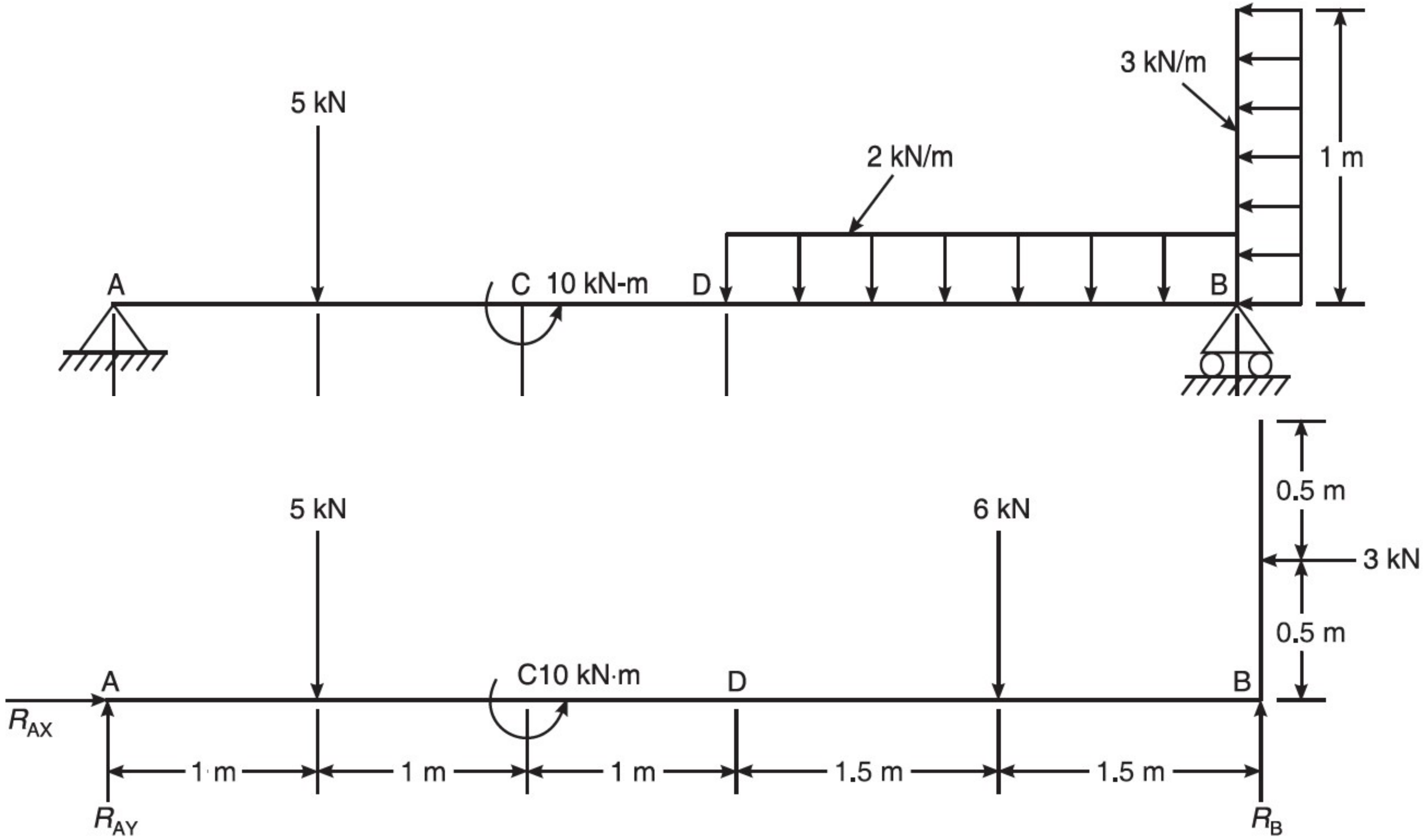


Using the conditions of equilibrium

$\Sigma F_x = 0$

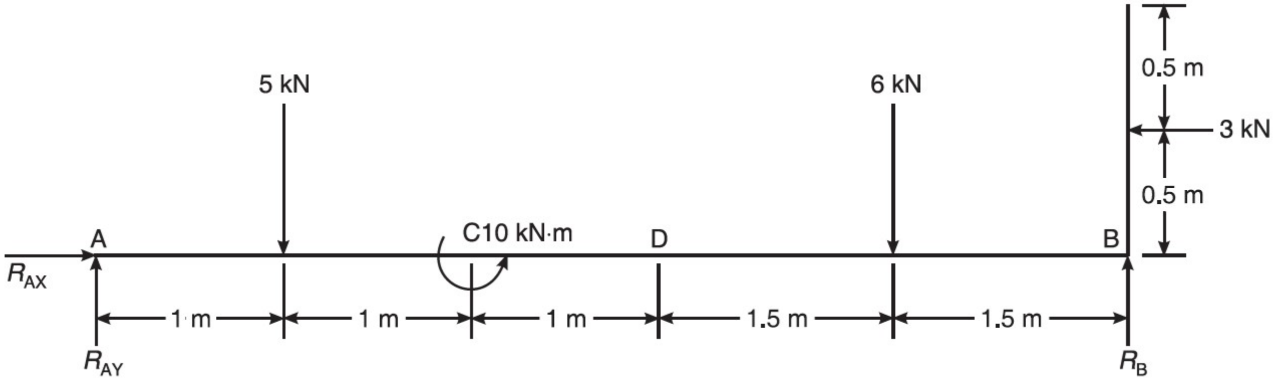
$R_{AX} – 3 = 0$

$R_{AX} = 3$ kN

$\Sigma F_Y = 0$

$R_{AY} + R_B -5-6 = 0$

$R_{AY} + R_B = 11$



$\Sigma M_A = 0$

$R_B \times 6 + 5 \times 1 + 6 \times 4.5 - 3 \times 0.5 - 10 = 0$

$R_B = 3.417$ kN

Substituting the value of R_B , we get

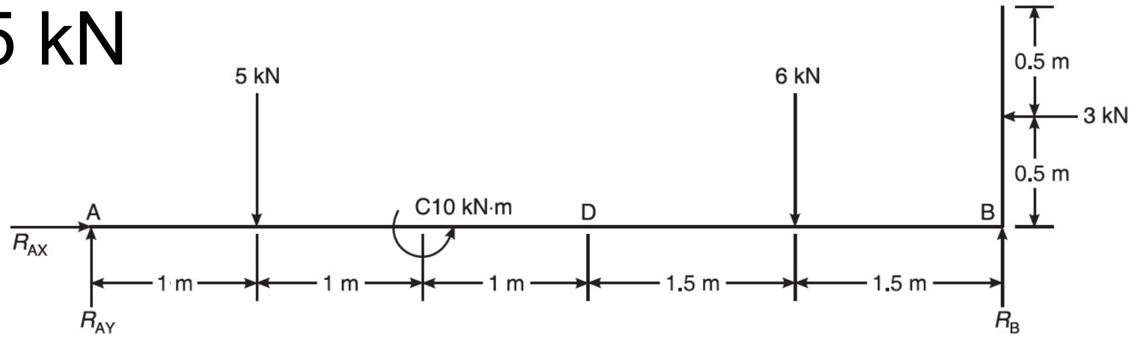
$R_{AY} + R_B = 11$

$R_{AY} + 3.417 = 11$

$R_{AY} = 7.583$ kN

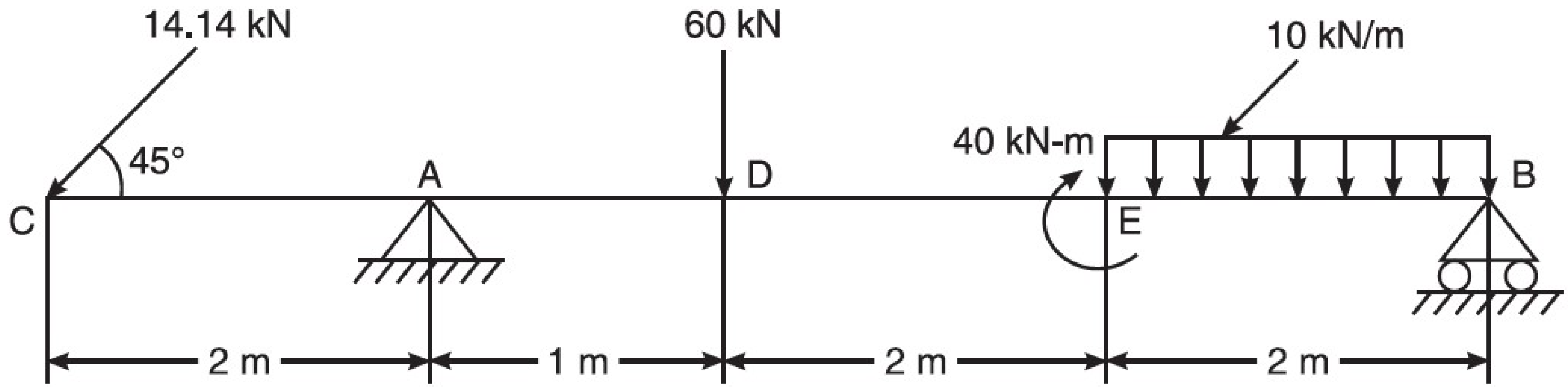
$$R_A = \sqrt{R_{AX}^2 + R_{AY}^2} = \sqrt{(3)^2 + (7.583)^2}$$

$R_A = 8.155$ kN



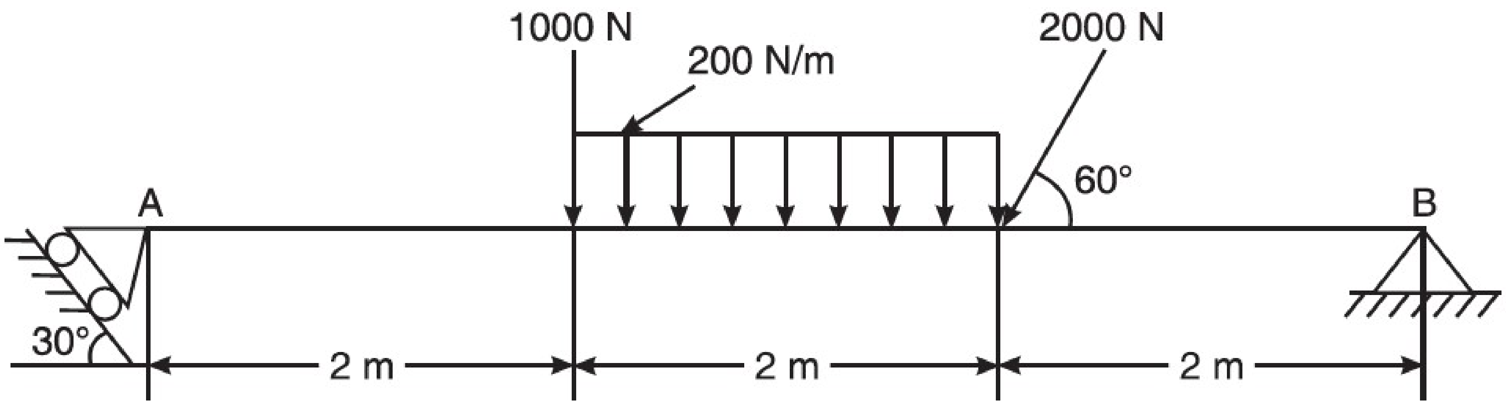
Comprehensive Question

1. Determine the support reactions for the beam supported and loaded as shown in Figure



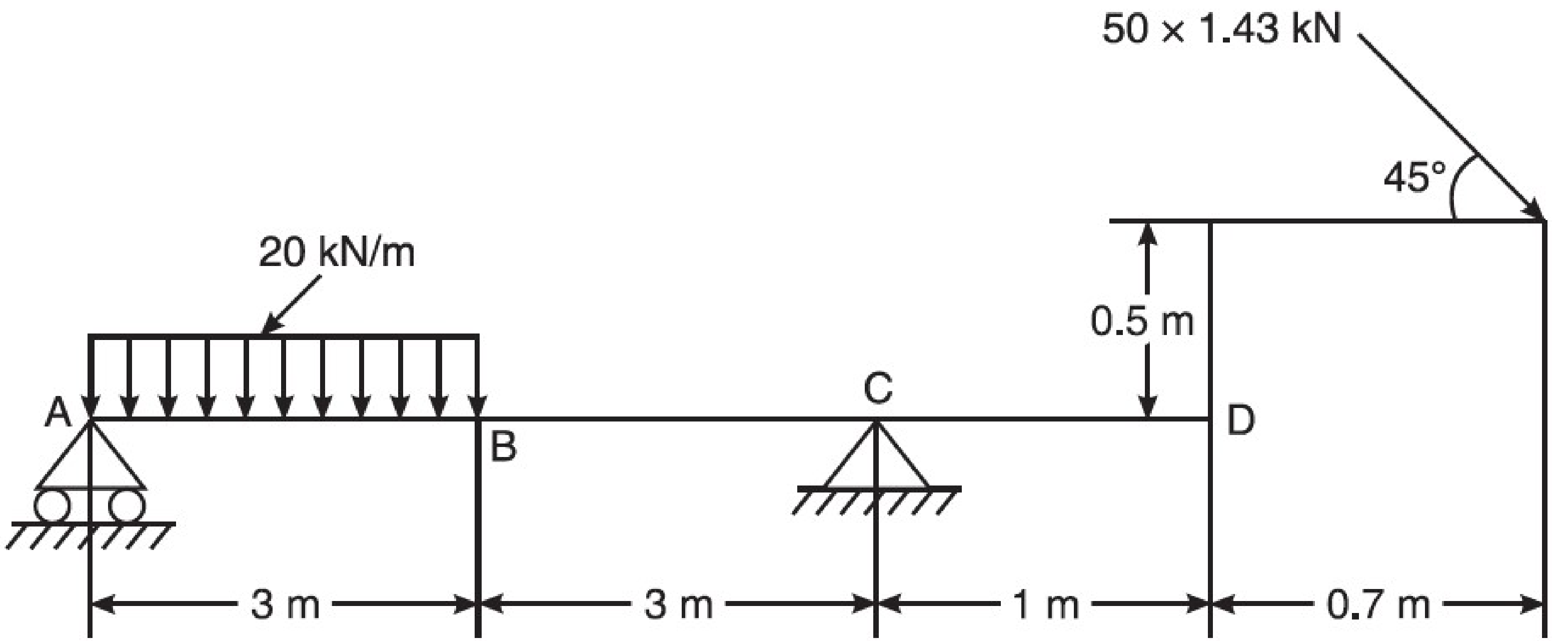
Ans: R_B = 32 kN, R_A = 58.85 kN

2. A horizontal beam 6 m long is supported on a knife edge at its end B and the end A, rests on a roller support placed on an inclined plane, having an inclination of 300 as shown in Figure. Find the reactions at the supports A and B.



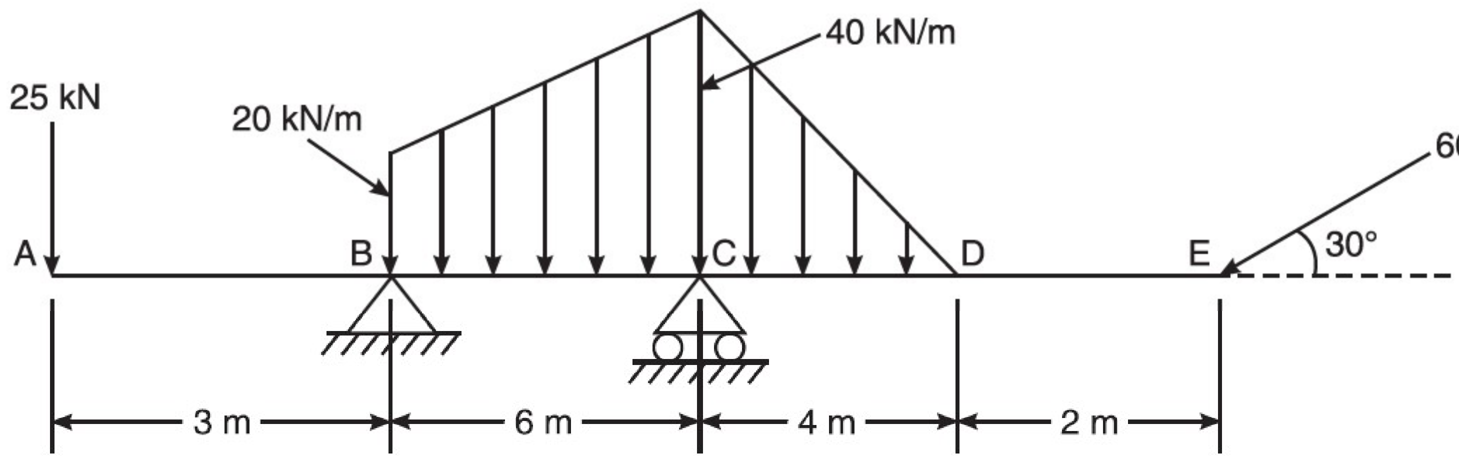
Ans: R_A = 1667.44 N, R_B = 1696.17 N

3. Compute the reactions at the supports of beam ABCD which is loaded and supported as shown in Figure.



Ans: R_{CX} = 50.558 kN, R_A = 26.462 kN, R_{CY} = 84.096 kN

4. Determine the reactions developed in the double overhanging beam shown in Figure.



Ans: R_{BX} = 51.962 kN, R_C = 245.273 kN, R_{BY} = 69.727 kN, R_B = 86.727 kN