



# INTRODUCTION TO CIVIL ENGINEERING

Prepared By:

Sagar C P

Assistant Professor

Civil Engineering Department

# Syllabus

## Centroid:

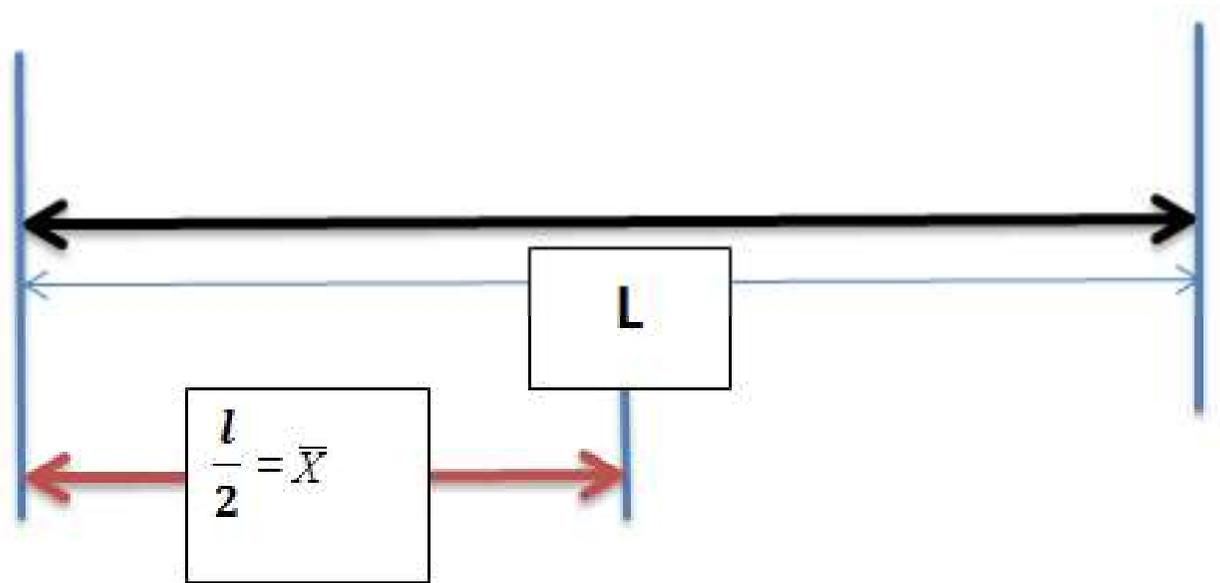
Importance of centroid and centre of gravity, determination of centroid by method of moments, axis of reference, methods of determining the centroid, Numerical examples for locating the centroid of built-up sections (Simple sections).

# CENTROID

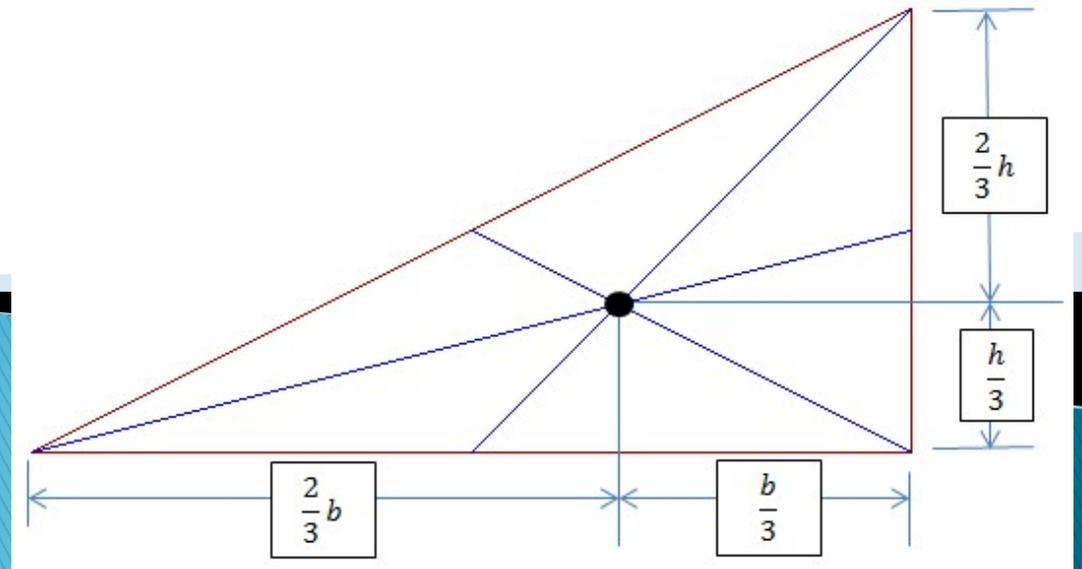
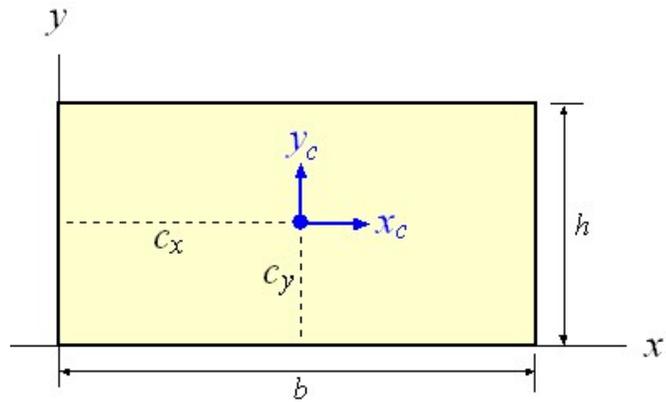
- ❖ Centroid is the geometric centre of the object
- ❖ The plane figures (like triangle, quadrilateral, circle etc.) have only areas, but no mass.
- ❖ The centre of area of such figures is known as **centroid**.
- ❖ Centroid of an area is a point at which the whole area of the plane figure is assumed to be concentrated.



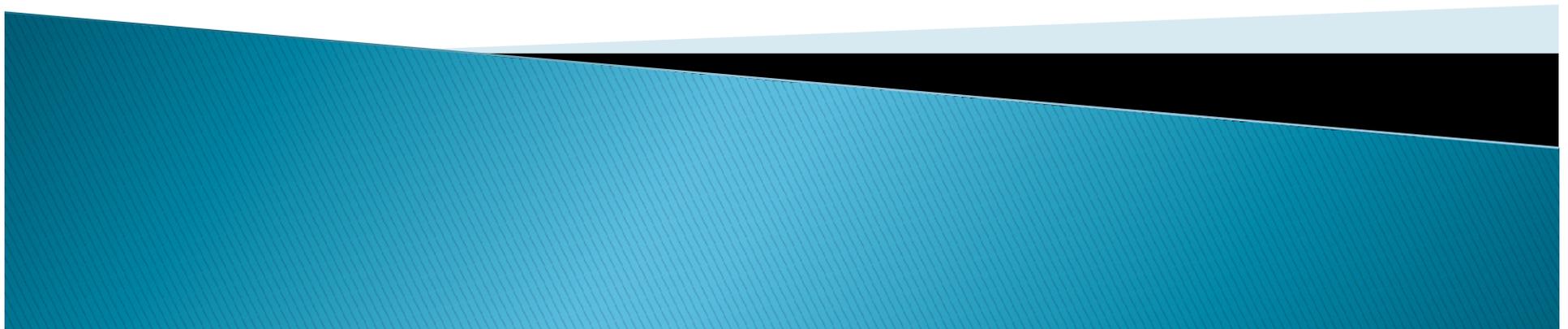
# Centroid of Line



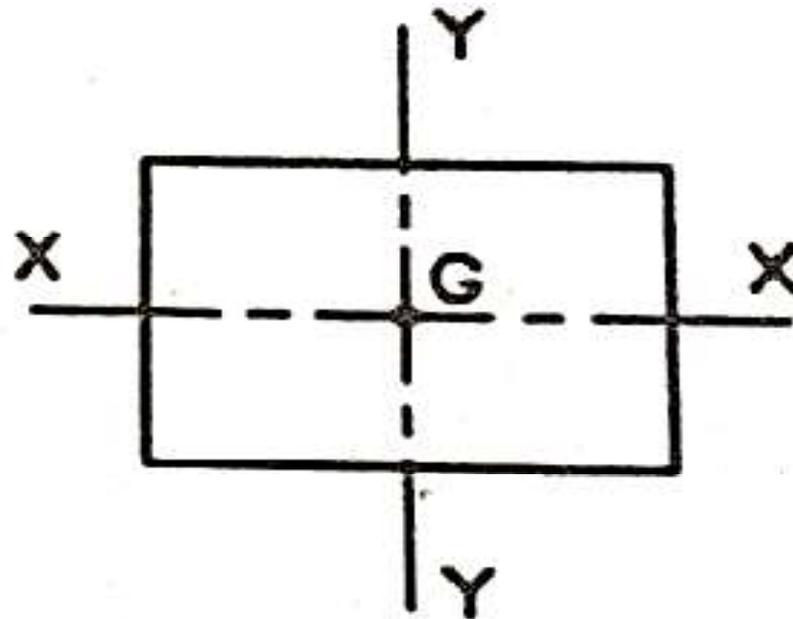
# Regular geometrical area



# Method of Integration

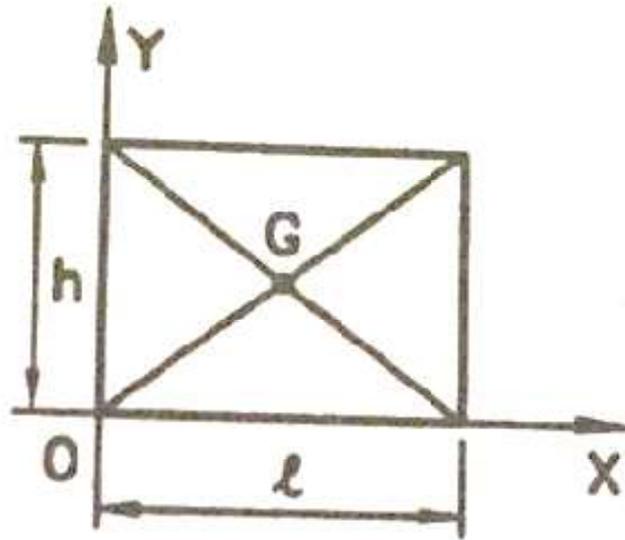


# Centroidal Axis



Source: Elements of Civil Engineering & Engineering Mechanics By M. N. Shesha Prakash, Ganesh B. Mogaveer

# Reference axis

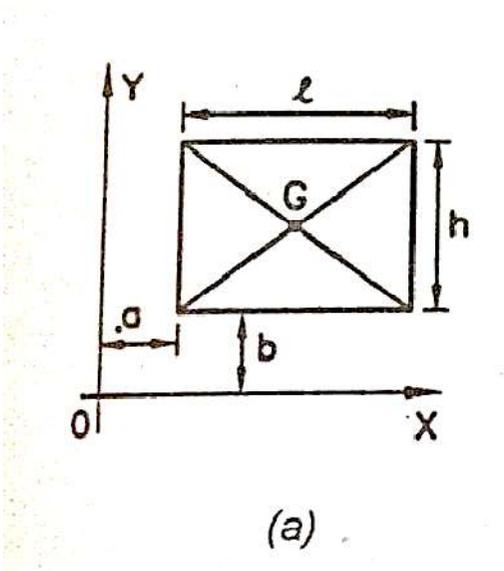


- ▶  $\bar{x} = l/2$
- ▶  $\bar{y} = h/2$

Source: Elements of Civil Engineering & Engineering Mechanics

By M. N. Shesha Prakash, Ganesh B. Mogaveer

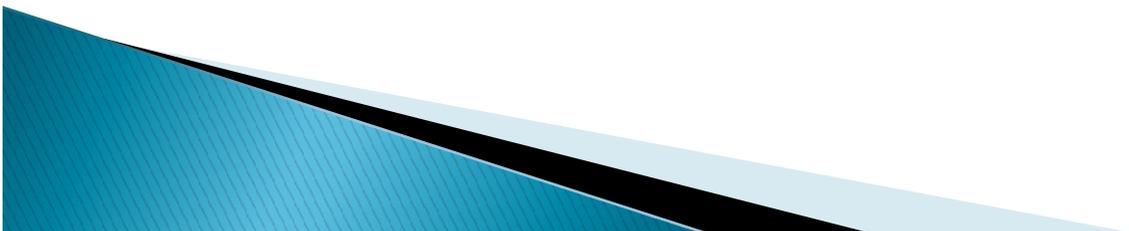
# Reference axis



$$\bar{x} = a + (l/2)$$
$$\bar{y} = b + (h/2)$$

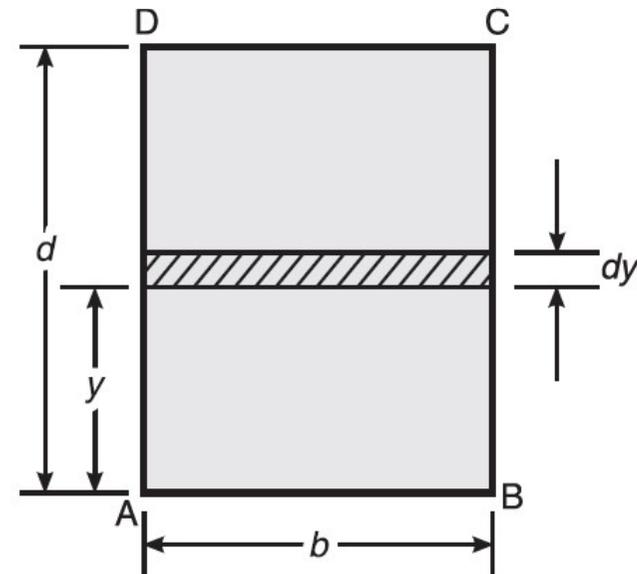
$$\bar{x} = (\sum ax) / (\sum a) \dots\dots (I)$$

$$\bar{y} = (\sum ay) / (\sum a) \dots\dots (II)$$



# Centroid of Rectangle by Integration Method

- ▶ Let us consider a rectangular lamina of area  $b \times d$  as shown in Figure. Now consider a horizontal elementary strip of area  $b \times dy$ , which is at a distance  $y$  from the reference axis AB.



Source: Elements of Civil Engineering & Engineering Mechanics  
By M. N. Shesha Prakash, Ganesh B. Mogaveer

Moment of area of elementary strip about AB =  $b \times dy \times y$

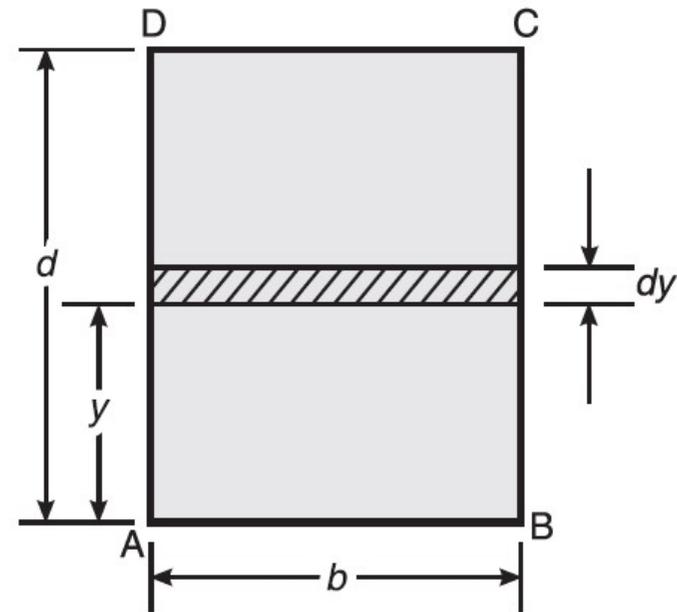
Sum of moments of such elementary strips about AB is given by

$$\int_0^d b \times dy \times y$$

$$= b \int_0^d y \cdot dy$$

$$= b \times \left[ \frac{y^2}{2} \right]_0^d$$

$$= \frac{bd^2}{2}$$



Moment of total area about AB =  $bd \times \bar{y}$   
Apply the principle of moments about AB

$$\frac{bd^2}{2} = bd \times \bar{y}$$

$$\bar{y} = \frac{d}{2}$$

By considering a vertical strip, similarly, we can prove that

$$\bar{x} = \frac{b}{2}$$



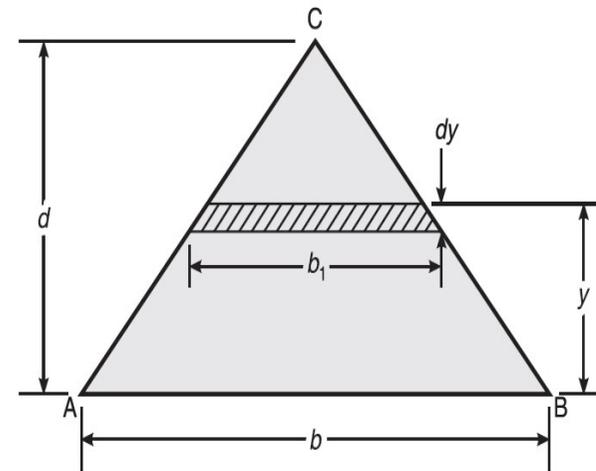
# Centroid of right angle triangle

Consider a triangular lamina of area  $(1/2) \times b \times d$  as shown in Figure

Now consider an elementary strip of area  $b_1 \times dy$  which is at a distance  $y$  from the reference axis AB. Using the property of similar triangles, we have

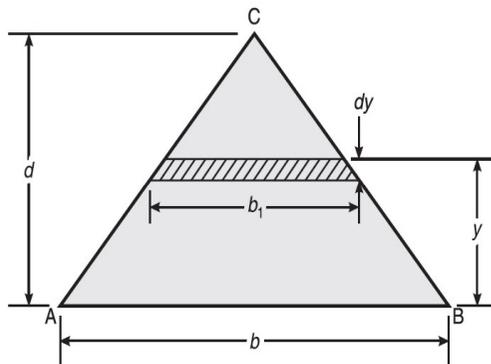
$$\frac{b_1}{b} = \frac{d - y}{d}$$

$$b_1 = \frac{(d - y)b}{d}$$



$$\text{Area of the elementary strip} = b_1 \times dy = \frac{(d - y)b \cdot dy}{d}$$

Moment of area of elementary strip about AB  
 = area  $\times$  y



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$$= \frac{(d - y)b \cdot dy \cdot y}{d}$$

$$= \frac{b \cdot dy \cdot d \cdot y}{d} - \frac{by^2 \cdot dy}{d}$$

$$= by \cdot dy - \frac{by^2 \cdot dy}{d}$$

Sum of moments of such elementary strips is given by

$$\begin{aligned} & \int_0^d by \cdot dy - \int_0^d \frac{by^2}{d} \cdot dy \\ &= b \times \left[ \frac{y^2}{2} \right]_0^d - \frac{b}{d} \left[ \frac{y^3}{3} \right]_0^d \\ &= \frac{bd^2}{2} - \frac{bd^3}{3d} \\ &= \frac{bd^2}{2} - \frac{bd^2}{3} \\ &= \frac{bd^2}{6} \end{aligned}$$

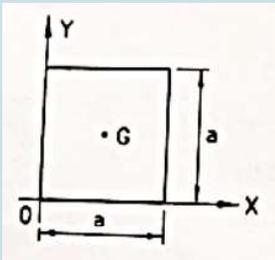
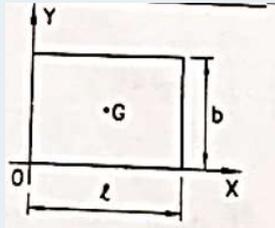
Moment of total area about AB  $= \frac{1}{2}bd \times \bar{y}$

Applying the principle of moments.

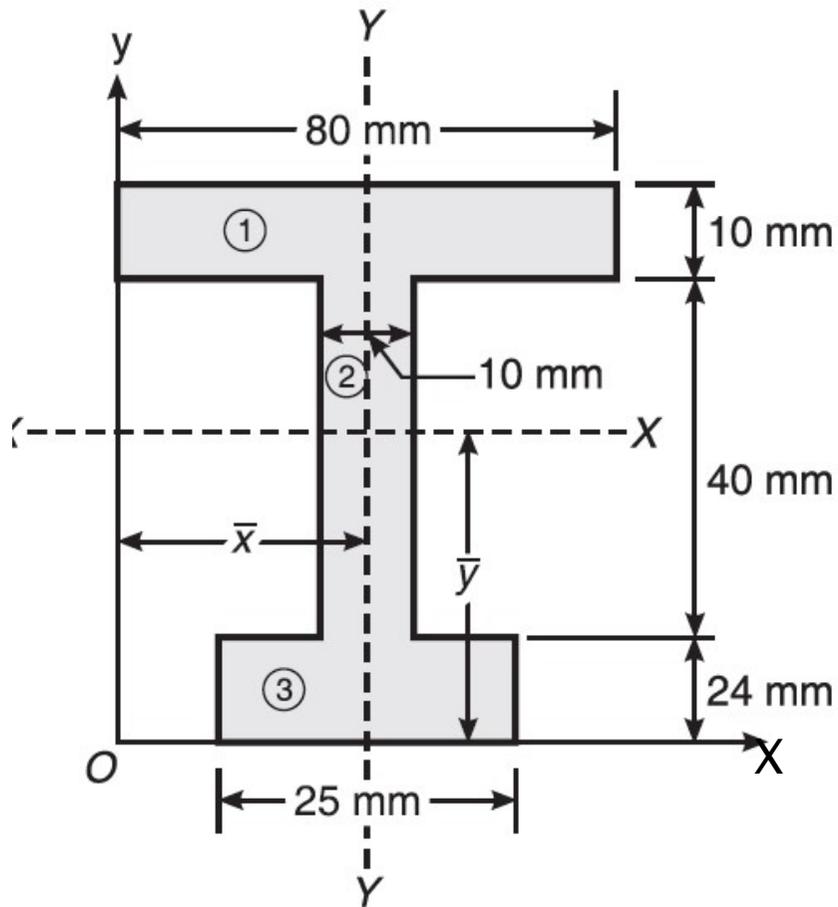
$$\frac{bd^2}{6} = \frac{1}{2} \times bd \times \bar{y}$$

$$\bar{y} = \frac{d}{3}$$

# Centroid of Simple Plane Figures

Name	Shape	$\bar{x}$	$\bar{y}$	Area
Square		$\frac{a}{2}$	$\frac{a}{2}$	$a^2$
Rectangle		$\frac{l}{2}$	$\frac{b}{2}$	$lb$

# 1) Find the centroid of Figure

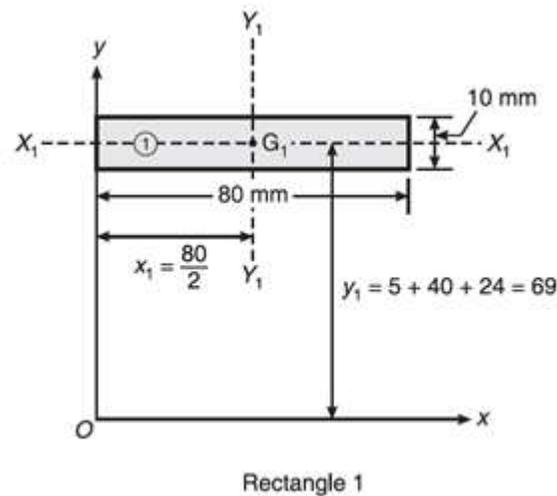
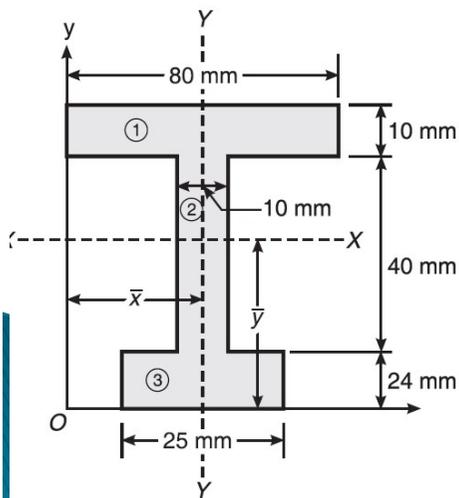


The given figure is symmetrical about the  $Y$ - $Y$  axis

$$\bar{x} = \frac{\text{Max horizontal distance}}{2}$$

$$\bar{x} = 80/2 = 40 \text{ mm}$$

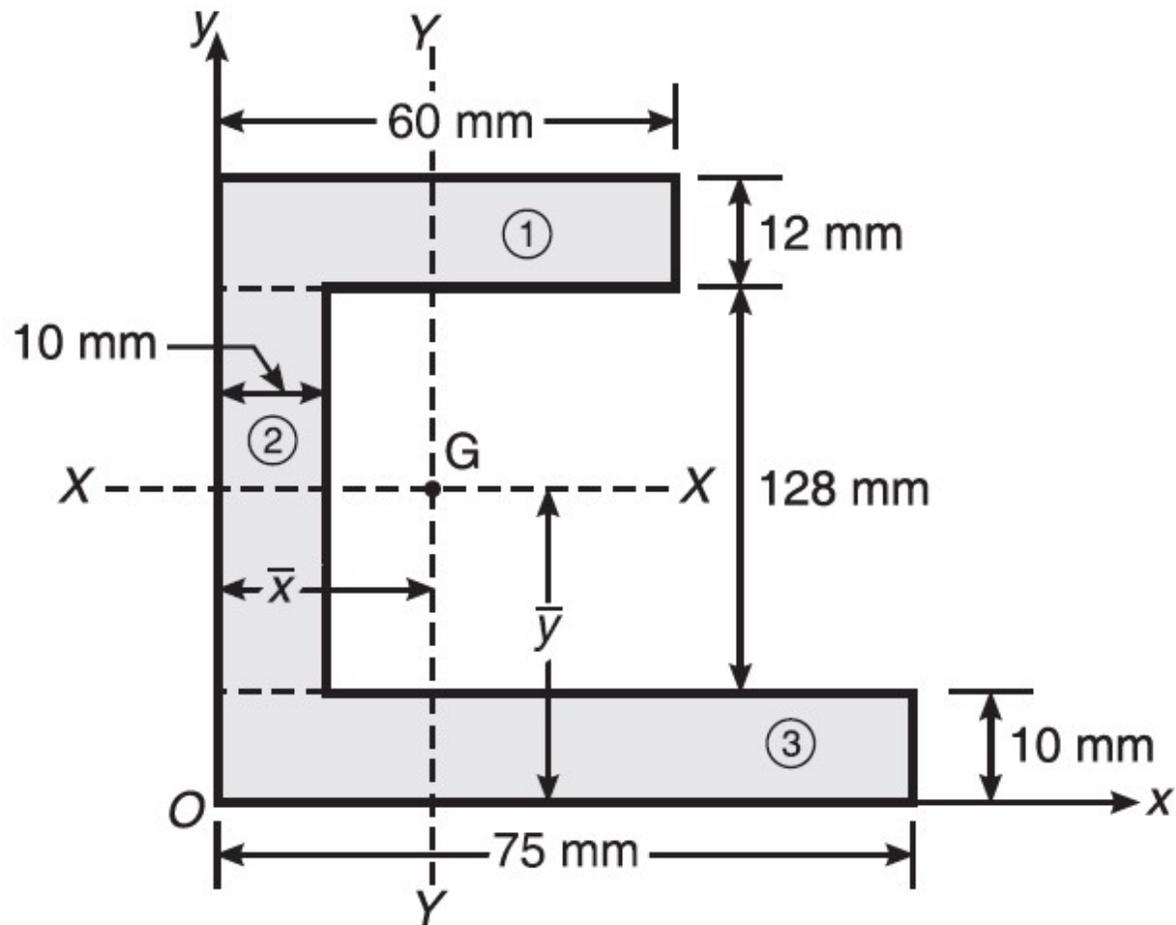
Component	Area a	Centroidal distance from the x- axis (y)	ay
Rectangle 1	$10 \times 80 = 800$	$24+40+(10/2) = 69$	55200
Rectangle 2	$10 \times 40 = 400$	$24+(40/2) = 44$	17600
Rectangle 3	$25 \times 24 = 600$	$(24/2) = 12$	7200
Sum	$\Sigma a = 1800$		$\Sigma ay = 80000$



$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{80,000}{1800} = 44.44 \text{ mm}$$

$$\bar{x} = \frac{80}{2} = 40 \text{ mm}$$

## 2) Determine the centroid of Figure

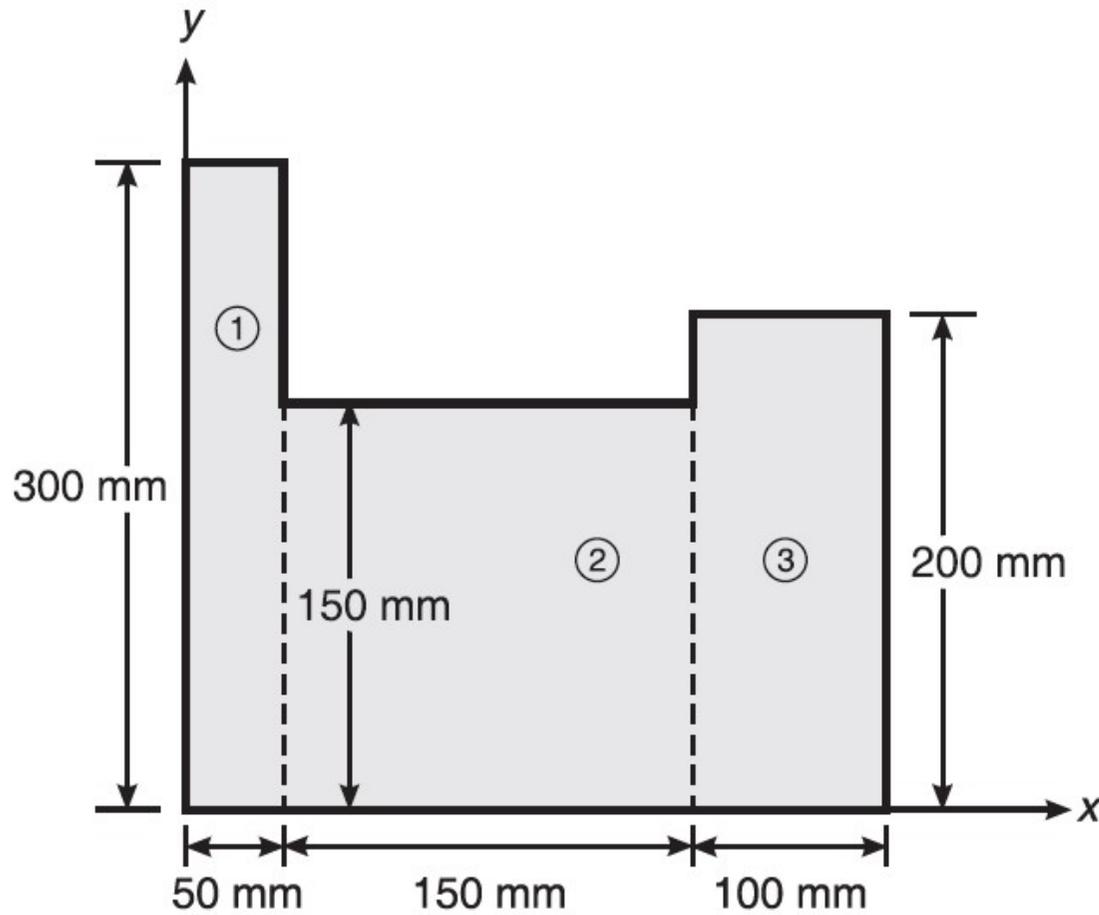


Component	Area a	Centroidal distance from the y-axis (x)	Centroidal distance from the x-axis (y)	ax	ay
Rectangle 1	60 x 12 = 720	60/2 = 30	10+128+(12/2) =144	21600	103680
Rectangle 2	10 x 128 = 1282	10/2 = 5	10+(128/2) =74	6400	94720
Rectangle 3	10 x 75 = 750	75/2 = 37.5	(10/2) = 5	28125	3750
Sum	$\Sigma a = 2750$			$\Sigma ax = 56125$	$\Sigma ay = 202150$

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{56,125}{2750} = 20.409 \text{ mm}$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{202,150}{2750} = 73.509 \text{ mm}$$

### 3) Determine the centroid of Figure



Component	Area a	Centroidal distance from the y- axis (x)	Centroidal distance from the x- axis (y)	ax	ay
Rectangle 1	50 x 300 = 15000	50/2 =25	(300/2)=150	37500	2250000
Rectangle 2	150 x 150= 22500	50+(150 /2) = 125	(150/2) =75	2812500	1687500
Rectangle 3	200 x 100= 20000	200+(10 0/2) = 250	(200/2) = 100	5000000	2000000
Sum	$\Sigma a = 57500$			$\Sigma ax = 8187500$	$\Sigma ay = 5937500$

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = \frac{8,187,500}{57,500} = 142.391 \text{ mm}; \quad \bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{5,937,500}{57,500} = 103.261 \text{ mm}$$