MALNAD COLLEGE OF ENGINEERING, HASSAN

DEPARTEMENT OF MATHEMATICS

Course Title	**Mathematics for Computer Science Engineering stream -1**		
Course Code	24MATS11	**(L-T-P)**	(3-1-2)4
Exam	3hours	**Hours / Week**	06
SEE	50 Marks	**Total Hours**	65(40L+13T+12P)

Course Objective: To train the students to acquire knowledge in calculus and numerical methods so as to solve basic engineering application problems.

Course Outcomes (COs): At the end of course, student will be able to:

COs	Outcomes	POs	PSOs
CO1	Compute Taylor series, partial derivatives, pedal equation, curvature and solve simple problems connected with multiple integrals, Counting principle, bayes theorem on probability, evolutes, errors and approximation.	PO1	-
CO2	Inspect for extreme values [the maximum output of a function (experimental data)] analyze the region of integration connected with multiple integrals so as to determine the area, volume. (remove add Probability)	PO1, PO2	-
CO3	Apply the numerical methods to compute: The area of a region, root (input) of an equation for the given output, missing in put or output of the given experimental data (interpolation/ extrapolation).	PO1	-
CO4	Model the real-life problems/engineering application problems and solve the same.	PO1, PO2	-
CO5	Write the program in python for the mathematical procedures connected with calculus, numerical methods, differential equations, vector calculus, execute the same and provides correct output.	PO1, PO2,PO5	-

MODULE –1	**10 Hrs.**

Differential Calculus: Definition of average growth rate and its illustrative examples. Definition of differentiability, Statement of Taylor's theorem, Taylor's series for a function of one variable - Illustrative examples. Polar coordinates, Polar curves, angle between the radius vector and the tangent, angle between two curves, Pedal equations,Curvature and Radius of curvature in polar and pedal form.

Applications : Extreme values of a single variable- cost and revenue.

Self Study: Brief introduction to evolutes and involutes. Indeterminate forms - L'Hospital's rule problems. Extreme values of a single variable. Expansion of a function as a Maclaurin's series for function of one variable.

Definition of average Growth Rate

In differential calculus, the average growth rate [or average rate of change] of a function over an interval $[a, b]$ is the change in the function output values divided by the change in input values.

Mathematically it is given by

$$\text{Average Growth Rate} = \frac{f(b) - f(a)}{b - a}$$

Here

* $f(a)$ & $f(b)$ are the funⁿ values at a & b.
* $b - a$ is the length of the interval.

Examples

1] Find the average growth rate of $f(x) = 2x + 3$ over the interval $[1, 4]$.

Sol: let $f(x) = 2x + 3$

Here $a = 1$, $b = 4$

$f(1) = 5, \quad f(4) = 11$

$$\text{Average Growth Rate} = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{f(4) - f(1)}{4 - 1}$$

$$= \frac{11 - 5}{3}$$

Average growth Rate $= 2$

Thus, the average growth rate of $f(x) = \frac{\cancel{x^2}}{2x+3}$ over the interval $[1, 4]$ is $\underline{\underline{2}}$.

2] Find the average growth rate of $f(x) = x^2$ over the interval $[2, 5]$

Sol. Let $f(x) = x^2$

Here $a = 2, \quad b = 5$

$f(2) = 4, \quad f(5) = 25$

Average growth rate $= \dfrac{f(b) - f(a)}{b-a}$

$$= \frac{f(5) - f(2)}{5-2}$$

$$= \frac{25-4}{3}$$

$$= \frac{21}{3}$$

$$= 7$$

Thus the average growth rate of $f(x) = x^2$ over the interval $[2, 5]$ is $\underline{7}$.

3) Find the average growth rate of $f(x) = e^x$ over the interval $[0, 1]$.

Let $f(x) = e^x$

Here $a = 0$, $b = 1$

$f(0) = e^0 = 1$, $f(1) = e^1 = [\approx 2.718]$

Average growth rate $= \dfrac{f(b) - f(a)}{b-a}$

$$= \frac{f(1) - f(0)}{1 - 0}$$

$$= \frac{e - 1}{1}$$

$$\simeq 2.718 - 1$$

$$= 1.718$$

Thus, the average growth rate of $f(x) = e^x$ over the interval $[0, 1]$ is $\underline{\underline{1.718}}$.

Taylor's Series

A function $f(x)$ is defined at the Point $x = a$. Then taylor's series formula is given by

$$f(x) = f(a) + \frac{(x-a)}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} f'''(a) + \cdots + \frac{(x-a)^n}{n!} f^n(a).$$

Where $f'(x), f''(x), f'''(x) \ldots$ are differentiable at a point $x=a$.

Examples

I] obtain the Taylors Series of $\log_e x$ about $x=1$ up to the term containing 4th degree & hence ~~find~~ obtain $\log_e(1.1)$.

Sol: $f(x) = f(a) + \frac{(x-a)}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + \cdots$

let $f(x) = \log_e x$, $x=1$

$x = a = 1$

$$f(x) = f(1) + (x-1) f'(1) + \frac{(x-1)^2}{2} f''(1) + \frac{(x-1)^3}{6} f'''(1) + \frac{(x-1)^4}{24} f^{iv}(1) \rightarrow (1)$$

$f(x) = \log_e x \rightarrow (2) \Rightarrow f(1) = \log 1 = 0$

Diff (2) w.r.to x

$f'(x) = \frac{1}{x} \rightarrow (3) \Rightarrow f'(1) = \frac{1}{1} = 1$

Diff ③ w.r.to x

$$f''(x) = \frac{-1}{x^2} \rightarrow (4) \Rightarrow f''(1) = -1$$

Diff ④ w.r.to x

$$f'''(x) = \frac{2}{x^3} \rightarrow (5) \Rightarrow f'''(1) = 2$$

Diff ⑤ w.r.to x

$$f^{IV}(x) = \frac{-6}{x^4} \Rightarrow f^{IV}(1) = -6$$

Substitute all the values in Eqn ①

$$f(x) = 0 + (x-1)(1) + \frac{(x-1)^2}{2}(-1) + \frac{(x-1)^3}{6}(2) + \frac{(x-1)^4}{24}(-6)$$

At $x = 1.1$

$$\cancel{f(x)}\ \left\langle f(1.1) = 0.095 \right\rangle$$

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2) Expand $\tan^{-1}x$ in the power of $(x-1)$ up to the term containing 3rd degree.

Sol: $f(x) = f(a) + \frac{(x-a)}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} f'''(a) + \cdots \rightarrow (1)$

$f(x) = \tan^{-1}x, \quad x = 1 = a$

$f(x) = \tan^{-1}x \quad \Rightarrow f(1) = \tan^{-1}(1) = \pi/4$

$f'(x) = \frac{1}{1+x^2} \quad \Rightarrow f'(1) = \frac{1}{2} = 0.5$

$f''(x) = \frac{-1}{(1+x^2)^2} \times 2x \Rightarrow f''(1) = \frac{-1}{(1+1)^2} \times 2(1)$

$= \frac{-2}{4} = \frac{-1}{2}$

$(1+x^2)^2 f''(x) = -2x$

Again diff w.r.to x. we get

$f'''(1) = 1/2$

$(1) \Rightarrow$

$f(x) = \frac{\pi}{4} + (x-1)\left(\frac{1}{2}\right) + \left(\frac{-1}{2}\right)\left(\frac{(x-1)^2}{2}\right) + \frac{1}{2}\frac{(x-1)^3}{6}$

$f(x) = \frac{\pi}{4} + \frac{x-1}{2} - \frac{(x-1)^2}{4} + \frac{(x-1)^3}{12}$

3] Obtain Taylor's series expansion of $\log(\cos x)$ about the point $x = \pi/3$ up to ~~the~~ 3rd degree term.

Sol: $f(x) = f(a) + \frac{(x-a)}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} f'''(a) + \ldots$

Let $f(x) = \log(\cos x)$, $x = \pi/3 = a$

$$f(x) = f\left(\frac{\pi}{3}\right) + \left(x - \frac{\pi}{3}\right) f'\left(\frac{\pi}{3}\right) + \frac{\left(x - \frac{\pi}{3}\right)^2}{2} f''(\pi/3) + \ldots \quad \to (1)$$

Consider

$f(x) = \log[\cos x] \Rightarrow f(\pi/3) = \log\left[\cos \frac{\pi}{3}\right]$

$= \log(1/2) = -\log 2$

$f'(x) = \frac{1}{\cos x}[-\sin x] \Rightarrow f'(x) = -\tan x$

$f'(\pi/3) = -\sqrt{3}$

$f''(x) = -\sec^2 x \Rightarrow f''\left(\frac{\pi}{3}\right) = \left(1 + \tan^2 \frac{\pi}{3}\right)$

$f''\left(\frac{\pi}{3}\right) = -4$

$$f'''(x) = -2\sec^2 x \cdot \tan x$$

$$f'''\left(\frac{\pi}{3}\right) = -2\sec^2\left(\frac{\pi}{3}\right)\cdot\tan\left(\frac{\pi}{3}\right)$$

$$= -2(4)(\sqrt{3})$$

$$f'''(\pi/3) = -8\sqrt{3}$$

From ①

$$f(x) = -\log 2 - \left(x-\frac{\pi}{3}\right)(\sqrt{3}) - 2\left(x-\frac{\pi}{3}\right)^2 - \frac{4}{\sqrt{3}}\left(x-\frac{\pi}{3}\right)^3 + \cdots$$

$\boxed{4}$ Find the Taylor's series for $\log(1+\cos x)$ at $x=\pi/2$ up-to 3^{rd} degree.

Sol: $f(x) = f(a) + \frac{(x-a)}{1!}f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a) + \cdots$

$$f(x) = \log(1+\cos x) \Rightarrow f\left(\frac{\pi}{2}\right) = \log\left(1+\cos\frac{\pi}{2}\right) = 0$$

$$\log(1)$$

$$f'(x) = \frac{1}{1+\cos x}(-\sin x) \Rightarrow f'\left(\frac{\pi}{2}\right) = -1$$

$$(1+\cos x)f'(x) = -\sin x$$

$$(1+\cos x)f''(x)+f'(x)(-\sin x)=-\cos x$$

$$\left(1+\cos\frac{\pi}{2}\right)f''\left(\frac{\pi}{2}\right)+f'\left(\frac{\pi}{2}\right)\left(-\sin\frac{\pi}{2}\right)=-\cos\frac{\pi}{2}$$

$$(1+0)f''\left(\frac{\pi}{2}\right)-1(-1)=0$$

$$\left\langle f''\left(\frac{\pi}{2}\right)=-1\right\rangle$$

$$f(x)=-\left(x-\frac{\pi}{2}\right)-\frac{\left(x-\frac{\pi}{2}\right)^2}{2}-\frac{\left(x-\frac{\pi}{2}\right)^3}{6}$$

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5. obtain Taylor's series expansion of $\log(1+e^x)$ at $x=0$ upto considering 3rd degree.

Let $f(x)=\log(1+e^x)$ $\quad x=a=0.$

Taylor's series formula.

$$f(x)=f(a)+\frac{(x-a)}{1!}f'(a)+\frac{(x-a)^2}{2!}f''(a)+\frac{(x-a)^3}{3!}f'''(a)+\text{-----}$$

$$f(x)=f(0)+\frac{(x-0)}{}f'(0)+\frac{(x-0)^2}{2}f''(0)+\frac{(x-0)^3}{3}f'''(0)+ \text{---} \rightarrow(1)$$

$$f(x)=\log(1+e^x) \rightarrow(2)$$

$$=\log(1+e^0)$$

$$=\log(1+1)$$

$$f(0)=\log 2$$

using eq^n (2) diff w.r.t 'x'

$$f(a)=\frac{1}{1+e^x}+\frac{1}{x}xe^x \rightarrow(3)$$

$$=\frac{1}{1+e^x}+\frac{1}{0}xe^0$$

$$=\frac{1}{1+e^x}$$

$$\Rightarrow \frac{1}{1+1}=\frac{1}{2}$$

$$\boxed{f'(0)=1/2}$$

using eq^n (3) diff w.r.t 'x'

$$(1+e^x)f''(x)+e^x f'(x)=e^x \rightarrow(4)$$

$$(1+e^0)f''(0)+e^0 f'(1/2)=e^0$$

$$1+1 \quad f''(0)+1\ (1/2)=1$$

$$2f''(0)+1/2=1$$

$$2f''(0)=1-1/2$$

$$2f''(0)=1/2 \qquad \boxed{f''(0)=1/4}$$

using eq^n (4) diff w.r.t 'x'

$(1+e^x) f'''(x) + e^x f''(x) + e^x f''(x) + e^x f'(x) = e^x$

$(1+1) f'''(0) + 1(1/4) + 1.(1/4) + 1(1/2) = 1$

$2f'''(0) + 1/4 + 1/4 + 1/2 = 1$

$2f'''(0) + 1 = 1$

$2f'''(0) = 1-1$

$\boxed{f'''(0) = 0}$

substitute all these value in eq^n (1)

$f(x) = \log 2 + (x-0) 1/2 + \frac{(x-0)^2}{2} (1/4) + \frac{(x-0)^3}{36} (0) + - - - -$

$= \log 2 + \frac{(x-0)}{2} + \frac{(x-0)^2}{8} - - -$

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6. obtain Taylor's series expansion of $\sin x$ at $x = \pi/2$ upto considering 3rd degree

let $f(x) = \sin x \qquad x = a = \pi/2$.

Taylor's series expansion formula.

$f(x) = f(a) + \frac{(x-a)}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} f'''(a) + - - - -$

$f(x) = f(\pi/2) + (x-\pi/2) f'(\pi/2) + \frac{(x-\pi/2)^2}{2} f''(\pi/2) + \frac{(x-\pi/2)^3}{3} f'''(\pi/2) + - - \rightarrow (1)$

$f(x) = \sin x \rightarrow (2)$

$f(\pi/2) = \sin \pi/2$

$f(\pi/2) = 1.$

using eq^n (2) diff w.r.t 'x'

$f'(x) = \cos x \rightarrow (3)$

$f'(\pi/2) = \cos \pi/2$

$f'(\pi/2) = 0$

using eq^n (3) diff w.r.t 'x'

$f''(x) = -\sin x \rightarrow (4)$

$f''(\pi/2) = -\sin(\pi/2)$

$f''(\pi/2) = -1.$

using eq^n (4) diff w.r.t 'x'

$f'''(x) = -\cos x$

$f'''(\pi/2) = -\cos \pi/2$

$f'''(\pi/2) = 0$

substitute all these value in eq^n (1)

$f(x) = 1 + (x-\pi/2) 0 + \frac{(x-\pi/2)^2}{2} (-1) + \frac{(x-\pi/2)^3}{36} (0) + - - -$

$= 1 - \frac{(x-\pi/2)^2}{2}$

7. Obtain Taylor's series expansion of $\cos 2x$ at $x = \pi/2$ upto considering 3rd degree

Solu: Let $f(x) = \cos 2x$ $\quad x = a = \pi/2$.

Taylor's series formula.

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a) + \cdots$$

$$f(x) = f(\pi/2) + (x-\pi/2)f'(\pi/2) + \frac{(x-\pi/2)^2}{2}f''(\pi/2) + \frac{(x-\pi/2)^3}{3}f'''(\pi/2) + \cdots \rightarrow (1)$$

$f(x) = \cos 2x \rightarrow (2)$

$f(\pi/2) = \cos 2(\pi/2)$

$f(\pi/2) = \cos(\pi)$

$f(\pi/2) = -1.$

using eq^n (2) diff w.r.t 'x'

$f'(x) = -2\sin 2x \rightarrow (3)$

$f'(\pi/2) = -2\sin \cancel{2}(\pi/\cancel{2})$

$f'(\pi/2) = -2(0)$

$f'(\pi/2) = 0.$

using eq^n (3) diff w.r.t 'x'

$f''(x) = -2.2\cos 2x \rightarrow (4)$

$f''(\pi/2) = \cancel{-2.2\cos} -4\cos \cancel{2}(\pi/\cancel{2})$

$f''(\pi/2) = -4$

using eq^n (4) diff w.r.t 'x'

$f'''(x) = +4.2\sin x$

$f'''(\pi/2) = 8\sin x(\pi/2)$

$f'''(\pi/2) = -8.\sin \cancel{2}(\pi/\cancel{2})$

$f'''(\pi/2) = 0.$

Substitude all these value in eq^n (1).

$$f(x) = -1(x-\pi/2)0 + \frac{(x-\pi/2)^2}{2}\cancel{4}^2 + \frac{(x-\pi/2)^3}{6}0 + \cdots$$

$$f(x) = -1 + (x-\pi/2)^2 2$$

8. Obtain Taylor's series expansion of $\sin x + \cos x$ at $x = \pi/2$ upto 4th degree.

Solu: Let $f(x) = \sin x + \cos x$ $\quad x = a = \pi/2$

Taylor's series formula.

$$f(x) = f(a) + \frac{(x-a)}{1!}f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a) + \frac{(x-a)^4}{4!}f^{iv}(a) + \cdots$$

$$f(x) = f(\pi/2) + (x-\pi/2)f'(\pi/2) + \frac{(x-\pi/2)^2}{2}f''(\pi/2) + \frac{(x-\pi/2)^3}{3}f'''(\pi/2) + \frac{(x-\pi/2)^4}{4}f^{iv}(\pi/2) + \cdots \rightarrow (1)$$

$f(x) = \sin x + \cos x \rightarrow (2)$

$f(\pi/2) = \cos x - \sin x$

$f(\pi/2) = \cos(\pi/2) - \sin(\pi/2)$

$f(\pi/2) = \sin \pi/2 + \cos \pi/2$

$f(\pi/2) = 1 + 0$

$f(\pi/2) = 1$

using eq^n (2) diff w.r.t 'x'

$f'(x) = \cos x - \sin x \rightarrow (3)$

$f'(x) = \cos(\pi/2) - \sin(\pi/2)$

$f'(x) = 0 - 1$

$f'(x) = -1.$

using eq^n (3) diff w.r.t 'x'

$f''(x) = -\sin x - \cos x \rightarrow (4)$

$f''(x) = -\sin(\pi/2) - \cos(\pi/2)$

$f''(x) = -1 - 0$

$f''(x) = -1$

using eq^n (4) diff w.r.t 'x'

$f'''(x) = -\cos x + \sin x \rightarrow (5)$

$f'''(x) = -\cos(\pi/2) + \sin(\pi/2)$

$f'''(x) = -0 + 1$

$f'''(x) = 1.$

using eq^n (5) diff w.r.t 'x'

$f^{IV}(x) = \sin x + \cos x$

$f^{IV}(\pi/2) = \sin \pi/2 + \cos \pi/2$

$f^{IV}(\pi/2) = 1 + 0$

$f^{IV}(\pi/2) = 1.$

Substitude all these value in eq^n (1)

$$f(x) = f(1) + (x - \pi/2) \cdot -1 + \frac{(x-\pi/2)^2}{2} \cdot -1 + \frac{(x-\pi/2)^3}{3}(1) + \frac{(x-\pi/2)^4}{4} \cdot (1)$$

$$f(x) = 1 - (x - \pi/2) - \frac{(x-\pi/2)^2}{2} + \frac{(x-\pi/2)^3}{6} + \frac{(x-\pi/2)^4}{24} + \cdots$$

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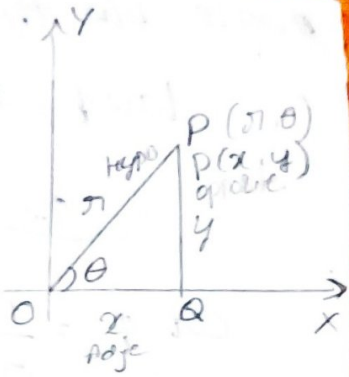
POLAR COORDINATES



Let (x.y) & (r.θ) represent the cartesian & polar coordinates of any point P in the plane. The origin 'O' is pole & x-axis is taken as the initial line.

From the fig. we have OQ = x & PQ = y. Also from right angle triangle OQP we have

$$\cos\theta = \frac{OQ}{OP} = \frac{\text{adjacent}}{\text{Hypotenuse}} = \frac{x}{r}$$

$$x = r\cos\theta \quad \text{— (1)}$$

$$\sin\theta = \frac{QP}{OP} = \frac{\text{opposite}}{\text{Hypotenuse}} = \frac{y}{r}$$

$$y = r\sin\theta \quad \text{— (2)}$$

Squaring & adding eq^n (1) & (2) we get

$$x^2+y^2 = r^2(\cos^2\theta + \sin^2\theta)$$

$$x^2+y^2 = r^2$$

$$r = \sqrt{x^2+y^2} \quad \text{— (3)}$$

Also ÷ (2) by (1)

$$\frac{r\sin\theta}{r\cos\theta} = \frac{y}{x}$$

$$\tan\theta = \frac{y}{x} \Rightarrow \theta = \tan^{-1}(y/x) \quad \text{— (4)}$$

Here eq^n (1) & (2) are the cartesian coordinates in terms of polar coordinates.

Eq^n (3) & (4) are the polar coordinates in terms of cartesian coordinates.

Angle b/w the radius vector and tangent

$$\tan\phi = r\left[\frac{d\theta}{dr}\right]$$

OR

$$\cot\phi = \frac{1}{r}\left[\frac{dr}{d\theta}\right] \quad \left[\because \frac{1}{\tan\phi} = \frac{1}{r}\left[\frac{dr}{d\theta}\right]\right]$$

Slope of the tangent

$$\psi = \phi + \theta$$

$$\tan\psi = \tan(\phi + \theta)$$

Pairs of curves intersect each other orthogonally.

$$|\phi_1 - \phi_2| = \frac{\pi}{2} \quad [\text{orthogonality condition}]$$

Angle of intersection.

$$|\phi_1 - \phi_2| \text{ or } |\phi_2 - \phi_1|$$

Working procedure

1] The eq^n in the form $r = f(\theta)$ then take log on both sides of the eq^n.

2] Diff w.r.t θ. then it gives the term $\frac{1}{r}\frac{dr}{d\theta}$

3] Then directly substitute $\cot\phi$ or $\cot\phi_1$ or $\cot\phi_2$.

4] Simplify R.H.S terms gives the angle of intersection $|\phi_2 - \phi_1|$ or $|\phi_1 - \phi_2|$

NOTE:

1] $\sin(\pi/2-\theta) = \cos\theta$ $\quad\quad$ $\cos(\pi/2-\theta) = \sin\theta$

$\tan(\pi/2-\theta) = \cot\theta$ $\quad\quad$ $\cot(\pi/2-\theta) = \tan\theta$

2] $\sin(\pi/2+\theta) = \cos\theta$ $\quad\quad$ $\cos(\pi/2+\theta) = -\sin\theta$

$\tan(\pi/2+\theta) = -\cot\theta$ $\quad\quad$ $\cot(\pi/2+\theta) = -\tan\theta$

3] $\tan(\pi/4+\theta) = \dfrac{1+\tan\theta}{1-\tan\theta}$, $\cot(\pi/4+\theta) = \dfrac{1-\tan\theta}{1+\tan\theta}$

4] $1+\cos\theta = 2\cos^2(\theta/2)$

$1-\cos\theta = 2\sin^2(\theta/2)$

5] $\sin\theta = 2\sin(\theta/2).\cos(\theta/2)$

6] $\cos\theta = \cos^2(\theta/2) - \sin^2(\theta/2)$

Problems

Find the angle b/w the radius vector & the tangent for the following curves

① $r = a(1-\cos\theta)$

Soln:. Taking log on both sides

$\log(ab) = \log a + \log b$

$\log r = \log[a(1-\cos\theta)]$

$\log r = \log a + \log(1-\cos\theta)$

Diff w.r.t θ we get

$\log x = 1/x$

$\log(1-\cos\theta)$ $\quad \frac{1}{1-\cos\theta}(\sin\theta)$

$$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{\sin\theta}{1-\cos\theta}$$

$$\cot\phi = \frac{2\sin(\theta/2).\cos(\theta/2)}{2\sin^2(\theta/2)}$$

$$\cot\phi = \frac{\cos(\theta/2)}{\sin(\theta/2)}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\cot\phi = \cot(\theta/2)$$

$$\boxed{\phi = \theta/2}$$

2] $r^2 \cos 2\theta = a^2$

Taking log on b.s

$\log a^n = n \log a$

$\log ab = \log a + \log b$

$$\log(r^2 \cos 2\theta) = \log a^2$$

$$\log r^2 + \log(\cos 2\theta) = 2\log a$$

$$2\log r + \log(\cos 2\theta) = 2\log a$$

$\frac{d}{dx} \log x = \frac{1}{x}$

Diff w.r.t. θ.

$$\frac{2}{r}\frac{dr}{d\theta} + \frac{1}{\cos 2\theta}(-2\sin 2\theta) = 0$$

$$\frac{2}{r}\frac{dr}{d\theta} = \frac{2\sin 2\theta}{\cos 2\theta}$$

$$\frac{1}{r}\frac{dr}{d\theta} = \frac{\sin 2\theta}{\cos 2\theta}$$

$$\cot\phi = \tan 2\theta$$

$[\because \tan\theta = \cot(\pi/2 - \theta)]$

$$\cot\phi = \cot(\pi/2 - 2\theta)$$

$$\phi = (\pi/2 - 2\theta)$$

3] $r^m = a^m(\cos m\theta + \sin m\theta)$

Soln :- Taking log on b.s

$$\log r^m = \log(a^m(\cos m\theta + \sin m\theta))$$

$$m \log r = \log a^m + \log(\cos m\theta + \sin m\theta)$$

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$m \log r = m \log a + \log(\cos m\theta + \sin m\theta)$

Diff w.r.t. θ

$\frac{m}{r}\frac{dr}{d\theta} = 0 + \frac{1}{\cos m\theta + \sin m\theta}(-m \sin m\theta + m \cos m\theta)$

$\frac{m}{r}\frac{dr}{d\theta} = \frac{m(\cos m\theta - \sin m\theta)}{(\cos m\theta + \sin m\theta)}$

$\cancel{m}\left[\frac{1}{r}\frac{dr}{d\theta}\right] = \frac{\cancel{m}(\cos m\theta - \sin m\theta)}{(\cos m\theta + \sin m\theta)}$

$\cot\phi = \frac{\cos m\theta - \sin m\theta}{\cos m\theta + \sin m\theta}$

$\div$ by Numerator & denomenator of RHS by $\cos m\theta$

$\cot\phi = \frac{\frac{\cos m\theta}{\cos m\theta} - \frac{\sin m\theta}{\cos m\theta}}{\frac{\cos m\theta}{\cos m\theta} + \frac{\sin m\theta}{\cos m\theta}}$

$\cot\phi = \frac{1 - \tan m\theta}{1 + \tan m\theta}$

$\cot\phi = \cot(\pi/4 + m\theta)$

$\phi = (\pi/4 + m\theta)$

4] $\frac{l}{r} = 1 + \cancel{e\cos\theta}\ e\cos\theta$

Solu: Taking log on b.s

$\log(l/r) = \log(1 + e\cos\theta)$ $\qquad \log(m/n) = \log m - \log n$

$\log l - \log r = \log(1 + e\cos\theta)$

Diff w.r.t. θ

$$0 - \frac{1}{r}\frac{dr}{d\theta} = \frac{1}{1+e\cos\theta}(-e\sin\theta)$$

$$+\left(\frac{1}{r}\frac{dr}{d\theta}\right) = +\left(\frac{e\sin\theta}{1+e\cos\theta}\right)$$

$$\cot\phi = \frac{e\sin\theta}{1+e\cos\theta}$$ [this can't be simplified $\phi = \cot^{-1}\left(\frac{e\sin\theta}{1+e\cos\theta}\right)$

$$\frac{1}{\tan\phi} = \frac{e\sin\theta}{1+e\cos\theta} \Rightarrow \tan\phi = \frac{1+e\cos\theta}{e\sin\theta}$$

$$\phi = \tan^{-1}\left(\frac{1+e\cos\theta}{e\sin\theta}\right)$$

Find the angle between the radius vector and tangent and also find the slope of the tangent as indicated for the following curves.

I $r = a(1+\cos\theta)$ at $\theta = \pi/3$

Soln: Taking log on b.s.

$$\log r = \log(a(1+\cos\theta))$$

$$\log r = \log a + \log(1+\cos\theta)$$

Diff w.r.t θ.

$$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{(1+\cos\theta)}(-\sin\theta)$$

$$\cot\phi = \frac{-\sin\theta}{1+\cos\theta}$$

$$= \frac{-2\sin(\theta/2)\cdot\cos(\theta/2)}{2\cos^2(\theta/2)}$$

$$\cot\phi = \frac{-\cancel{2}\sin(\theta/2)}{\cos(\theta/2)}$$

$$\cot\phi = -\tan(\theta/2) \qquad \because -\tan\theta = \cot(\pi/2+\theta)$$

$$\cot\phi = \cot\left(\pi/2 + \theta/2\right)$$

$$\phi = \pi/2 + \theta/2$$

~~Also we have~~ $\cancel{\psi = \theta + \phi}$

At $\theta = \frac{\pi}{3}$, $\cancel{\psi = }$ $\phi = \pi/2 + \frac{\pi}{6}$

$$\phi = \frac{2\pi}{3} \Rightarrow \phi = \frac{2\pi}{3} = 120^\circ$$

Also we have $\psi = \theta + \phi$

$$\psi = \frac{\pi}{3} + \frac{2\pi}{3}$$

$$\psi = \frac{3\pi}{3} \Rightarrow \psi = \pi$$

$$\psi = 180^\circ$$

$\therefore$ Slope of the tangent $\tan\psi = \tan 180^\circ$

$$\tan\psi = 0$$

2] $r\cos^2(\theta/2) = a$ at $\theta = \frac{2\pi}{3}$

Soln: Taking log on b.s

$$\log(r\cos^2(\theta/2)) = \log a$$

$$\log r + \log\cos^2(\theta/2) = \log a$$

$$\log r + 2\log\cos(\theta/2) = \log a.$$

Diff w.r.t θ.

$$\frac{1}{r}\frac{dr}{d\theta} + 2\cdot\frac{1}{\cos(\theta/2)}\left(-\tfrac{1}{2}\cdot\sin(\theta/2)\right) = 0$$

$$\frac{1}{r}\frac{dr}{d\theta} = \frac{-2\cdot{}^{1}/_{2}\ \sin(\theta/2)}{\cos(\theta/2)}$$

$$\cot\phi = \tan(\theta/2)$$

$$\cot\phi = \cot\left(\pi/2 - \theta/2\right)$$

$$\phi = \left(\pi/2 - \theta/2\right)$$

At $\theta = \frac{2\pi}{3}$

$$\phi = \frac{\pi}{2} - \frac{2\pi/3}{2}$$

$$\phi = \frac{\pi}{2} - \frac{\pi}{3} \Rightarrow \phi = \frac{\pi}{6} = 30^\circ$$

Also $\psi = \theta + \phi$

$$\psi = \frac{2\pi}{3} + \frac{\pi}{6} \Rightarrow \psi = \frac{5\pi}{6}$$

$$\psi = 150^\circ$$

$\therefore$ Slope of the tangent $\tan\psi = \tan 150^\circ = \frac{-1}{\sqrt{3}}$

3] $\frac{2a}{r} = 1 - \cos\theta$ at $\theta = \frac{2\pi}{3}$

Solu:- Taking log on b.s

$$\log(2a/r) = \log(1-\cos\theta)$$

$$\log 2a - \log r = \log(1-\cos\theta)$$

Diff w.s.t θ

$$0 - \frac{1}{r}\frac{dr}{d\theta} = \frac{1}{1-\cos\theta}(\sin\theta)$$

$$-\cot\phi = \frac{2\sin(\theta/2)\cdot\cos(\theta/2)}{2\sin^2(\theta/2)}$$

$$-\cot\phi = \cot(\theta/2)$$

$\phi = -\theta/2$

At $\theta = \frac{2\pi}{3}$

$$\phi = \frac{-2\pi/3}{2}$$

$$\phi = -\pi/3$$

Also $\psi = \theta + \phi$

$$\psi = \frac{2\pi}{3} - \frac{\pi}{3} \Rightarrow \psi = \frac{\pi}{3}$$

$\therefore$ Slope of the tangent $\tan\psi = \tan(60^\circ) = \sqrt{3}$

4] $r = a(1+\sin\theta)$ at $\theta = \pi/2$

Soln :- Taking log on b.s

$$\log r = \log(a(1+\sin\theta))$$

$$\log r = \log a + \log(1+\sin\theta)$$

Diff w.r.t. θ

$$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{1+\sin\theta}(\cos\theta)$$

$1+\sin\theta = (\sin\theta + \cos\theta)^2$

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$

$$\cot\phi = \frac{\cos^2(\theta/2) - \sin^2(\theta/2)}{\cos^2(\theta/2) + \sin^2(\theta/2) + 2\sin(\theta/2)\cos(\theta/2)}$$

$$\cot\phi = \frac{[\cos(\theta/2) - \sin(\theta/2)][\cos(\theta/2) + \sin(\theta/2)]}{[\cos(\theta/2) + \sin(\theta/2)]^2}$$

$$\cot\phi = \frac{\cos(\theta/2) - \sin(\theta/2)}{\cos(\theta/2) + \sin(\theta/2)}$$

$\div$ by numerator and denominator of RHS by $\cos(\theta/2)$

$$\cot\phi = \frac{\frac{\cos(\theta/2)}{\cos(\theta/2)} - \frac{\sin(\theta/2)}{\cos(\theta/2)}}{\cos(\theta/2)/\cos(\theta/2) + \sin(\theta/2)/\cos(\theta/2)}$$

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$$\cot\phi = \frac{1-\tan(\theta/2)}{1+\tan(\theta/2)}$$

$$\cot\phi = \cot\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$$

$$\phi = \frac{\pi}{4}+\frac{\theta}{2}$$

At $\theta = \frac{\pi}{2}$ $\quad \phi = \frac{\pi}{4}+\frac{\pi/2}{2} \Rightarrow \phi = \frac{\pi}{4}+\frac{\pi}{4}$

$$\phi = \frac{\pi}{2}$$

Also, $\psi = \theta + \phi$

$$\psi = \frac{\pi}{2}+\frac{\pi}{2}$$

$$\psi = \pi$$

$\therefore$ Slope of the tangent $\tan\psi = \tan\pi = 0$

Show that the following pairs of curves intersect each other orthogonally.

1) $r = a(1+\cos\theta)$ and $r = b(1-\cos\theta)$

Soln :- Taking log on b.s

$\log r = \log(a(1+\cos\theta))$ $\quad\quad$ $\log r = \log(b(1-\cos\theta))$

$\log r = \log a + \log(1+\cos\theta)$ $\quad\quad$ $\log r = \log b + \log(1-\cos\theta)$

Diff these w.r.t θ

$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{(1+\cos\theta)}(-\sin\theta)$ $\quad\quad$ $\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{(1-\cos\theta)}(\sin\theta)$

$\cot\phi_1 = \frac{-\sin\theta}{1+\cos\theta}$ $\quad\quad$ $\cot\phi_2 = \frac{\sin\theta}{1-\cos\theta}$

$\cot\phi_1 = \dfrac{-2\sin(\theta/2)\cos(\theta/2)}{2\cos^2(\theta/2)}$; $\cot\phi_2 = \dfrac{2\sin(\theta/2)\cos(\theta/2)}{2\sin^2(\theta/2)}$

$\cot\phi_1 = -\tan(\theta/2)$. $\cot\phi_2 = \cot(\theta/2)$

$\cot\phi_1 = \cot(\pi/2 + \theta/2)$ $\phi_2 = \theta/2$

$\phi_1 = \pi/2 + \theta/2$

$\therefore$ Angle of intersection $= |\phi_1 - \phi_2| = |\pi/2 + \theta/2 - \theta/2|$

$|\phi_1 - \phi_2| = \pi/2$

Hence the curves intersect orthogonally

2] $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$

Soln:- Taking log on b.s

$\log(r^n) = \log(a^n \cos n\theta)$. $\log(r^n) = \log(b^n \sin n\theta)$

$\log r^n = \log a^n + \log\cos n\theta$. $\log r^n = \log b^n + \log\sin n\theta$

$n\log r = n\log a + \log(\cos n\theta)$. $n\log r = n\log b + \log(\sin n\theta)$

Diff these w.r.t. θ

$\dfrac{n}{r}\dfrac{dr}{d\theta} = 0 + \dfrac{1}{\cos n\theta}(-n\sin n\theta)$. $\dfrac{n}{r}\dfrac{dr}{d\theta} = 0 + \dfrac{1}{\sin n\theta}(n\cos n\theta)$

$\cancel{n}\cot\phi_1 = -\cancel{n}\tan n\theta$. $\cancel{n}\cot\phi_2 = \cancel{n}\cot n\theta$

$\cot\phi_1 = \cot(\pi/2 + n\theta)$ $\phi_2 = n\theta$

$\phi_1 = (\pi/2 + n\theta)$

$\therefore |\phi_1 - \phi_2| = |\pi/2 + \cancel{n\theta} - \cancel{n\theta}| = \pi/2$

Hence the curves intersect orthogonally.

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3] $r^2 \sin 2\theta = a^2$ and $r^2 \cos 2\theta = b^2$

Soln: Taking log on b.s

$\log(r^2 \sin 2\theta) = \log a^2$ $\qquad$ $\log(r^2 \cos 2\theta) = \log b^2$

$\log r^2 + \log \sin 2\theta = \log a^2$ $\qquad$ $\log r^2 + \log \cos 2\theta = \log b^2$

$2\log r + \log(\sin 2\theta) = 2\log a$. $\qquad$ $2\log r + \log(\cos 2\theta) = 2\log b$

Diff these w.r.t. θ

$\frac{2}{r}\frac{dr}{d\theta} + \frac{1}{\sin 2\theta}(2\cos 2\theta) = 0$ $\qquad$ $\frac{2}{r}\frac{dr}{d\theta} + \frac{1}{\cos 2\theta}(-2\sin 2\theta) = 0$

$\cancel{2}\frac{1}{r}\frac{dr}{d\theta} = -\cancel{2}\frac{\cos 2\theta}{\sin 2\theta}$ $\qquad$ $\cancel{2}\frac{1}{r}\frac{dr}{d\theta} = \cancel{2}\frac{\sin 2\theta}{\cos 2\theta}$

$\cot\phi_1 = -\cot 2\theta$ $\qquad$ $\cot\phi_2 = \tan 2\theta$

$\cot\phi = \cot(-2\theta)$ $\qquad$ $\cot\phi_2 = \cot(\pi/2 - 2\theta)$

$\phi_1 = -2\theta$ $\qquad$ $\phi_2 = \pi/2 - 2\theta$

$\therefore |\phi_1 - \phi_2| = |-\cancel{2\theta} - \pi/2 + \cancel{2\theta}| = \pi/2$

Hence the curves intersect ~~ott~~ orthogonally.

4] $r = ae^{\theta}$ and $re^{\theta} = b$.

Soln: Taking log on b.s

$\log r = \log(ae^{\theta})$ $\qquad$ $\log(re^{\theta}) = \log b$

$\log r = \log a + \log e^{\theta}$ $\qquad$ $\log r + \log e^{\theta} = \log b$

$\log r = \log a + \theta\log e$ $\qquad$ $\log r + \theta\log e = \log b$

Diff these w.r.t. θ

$\frac{1}{r}\frac{dr}{d\theta} = 0 + 1$ $\qquad$ $\frac{1}{r}\frac{dr}{d\theta} + 1 = 0$

$\cot\phi_1 = 1$ $\qquad$ $\cot\phi_2 = -1$

$\phi_1 = \cot^{-1}(1)$ $\qquad\qquad$ $\phi_2 = \cos^{-1}(-1)$

$\phi_1 = \frac{\pi}{4}$ $\qquad\qquad$ $\phi_2 = \frac{-\pi}{4}$

$\therefore |\phi_1 - \phi_2| = \left|\frac{\pi}{4} + \frac{\pi}{4}\right| = \frac{\pi}{2}$

Hence the curves intersect orthogonally

Find the angle of intersection of the following pairs of curves

I $r^2 \sin 2\theta = 4$ and $r^2 = 16 \sin 2\theta$

Soln :- Taking log on b.s

$\log(r^2 \sin 2\theta) = \log 4$. $\qquad$ $\log r^2 = \log(16 \sin 2\theta)$

$2\log r + \log(\sin 2\theta) = \log 4$. $\qquad$ $2\log r = \log 16 + \log \sin 2\theta$

Diff these w.r.t θ.

$2 \cdot \frac{1}{r}\frac{dr}{d\theta} + \frac{1}{\sin 2\theta}(2\cos 2\theta) = 0$. $\qquad$ $2\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{\sin 2\theta}(2\cos 2\theta)$

$2 \cdot \cot\phi_1 = -2\cot 2\theta$ $\qquad$ $2\cot\phi_2 = +2\cot 2\theta$

$\phi_1 = -2\theta$ $\qquad\qquad$ $\phi_2 = 2\theta$

$\therefore$ Angle of intersection. $|\phi_1 - \phi_2| = |-2\theta - 2\theta|$

$|\phi_1 - \phi_2| = 4\theta$ — (1)

Now consider. $r^2 = \frac{4}{\sin 2\theta}$ and $r^2 = 16 \sin 2\theta$

$\therefore \frac{4}{\sin 2\theta} = 16 \sin 2\theta$

$4 = 16 \sin^2 2\theta \Rightarrow 1 = \frac{16 \sin^2 2\theta}{4}$

$4 \sin^2 2\theta = 1$

$\sin^2 2\theta = \frac{1}{4} \Rightarrow \sin 2\theta = \frac{1}{2}$

$2\theta = \sin^{-1}(\frac{1}{2})$

$2\theta = \pi/6$

$\Rightarrow \theta = \frac{\pi}{12}$

Substituting $\theta = \frac{\pi}{12}$ in eq (i)

$|\phi_1 - \phi_2| = 4\frac{\pi}{12}$

$|\phi_1 - \phi_2| = \frac{\pi}{3}$

$\therefore$ Angle of intersection is $\pi/3 = 60°$

2] $r = a(1-\cos\theta)$ and $r = 2a\cos\theta$

Soln:- Taking log on b.s

$\log r = \log(a(1-\cos\theta))$

$\log r = \log a + \log(1-\cos\theta)$

Diff w.r.t. θ

$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{1-\cos\theta}(\sin\theta)$

$\cot\phi_1 = \frac{2\sin(\theta/2)\cos(\theta/2)}{2\sin^2(\theta/2)}$

$\cot\phi_1 = \cot(\theta/2)$

$\phi = \theta/2$

$\log r = \log(2a\cos\theta)$

$\log r = \log 2a + \log\cos\theta$

$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{\cos\theta}(-\sin\theta)$

$\cot\phi_2 = \frac{-\sin\theta}{\cos\theta}$

$\cot\phi_2 = -\tan\theta$

$\cot\phi_2 = \cot(\pi/2 + \theta)$

$\phi_2 = \frac{\pi}{2} + \theta$

$\therefore |\phi_1 - \phi_2| = |\frac{\theta}{2} - \frac{\pi}{2} - \frac{\theta}{2}|$

$= |\cancel{\frac{\pi}{2}} - \frac{\theta}{2} - \frac{\pi}{2}|$

$|\phi_1 - \phi_2| = \frac{\pi}{2} + \frac{\theta}{2}$ —— (1)

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Now consider $r = a(1-\cos\theta)$ and $r = 2a\cos\theta$

$\therefore a(1-\cos\theta) = 2a\cos\theta$

$a - a\cos\theta = 2a\cos\theta$

$a = 3a\cos\theta$

$1 = 3\cos\theta$

$\theta = \cos^{-1}(1/3)$ Sub in eq^n (1)

$\therefore$ the angle of intersection

$|\phi_1 - \phi_2| = \frac{\pi}{2} + \frac{\cos^{-1}(1/3)}{2}$

3] $r = 6\cos\theta$ and $r = 2(1+\cos\theta)$

Solu:- Taking log on b.s

$\log r = \log(6\cos\theta)$

$\log r = \log 6 + \log(\cos\theta)$

Diff w.r.t θ.

$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{\cos\theta}(-\sin\theta)$

$\cot\phi_1 = -\tan\theta$

$\cot\phi_1 = \cot(\pi/2 + \theta)$

$\phi_1 = \pi/2 + \theta$

$\log r = \log(2(1+\cos\theta))$

$\log r = \log 2 + \log(1+\cos\theta)$

$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{1+\cos\theta}(-\sin\theta)$

$\cot\phi_2 = \frac{-2\sin(\theta/2)\cos(\theta/2)}{2\cos^2(\theta/2)}$

$\cot\phi_2 = -\tan(\theta/2)$

$\cot\phi_2 = \cot(\pi/2 + \theta/2)$

$\phi_2 = \frac{\pi}{2} + \theta/2$

$\therefore$ Angle of intersection $|\phi_1 - \phi_2| = \left|\cancel{\frac{\pi}{2}} + \theta - \cancel{\frac{\pi}{2}} - \frac{\theta}{2}\right|$

$|\phi_1 - \phi_2| = \frac{\theta}{2}$ — (1)

Now consider $r = 6\cos\theta$ and $r = 2(1+\cos\theta)$

$\therefore 6\cos\theta = 2(1+\cos\theta)$

$6\cos\theta = 2 + 2\cos\theta$

$6\cos\theta - 2\cos\theta = 2$

$4\cos\theta = 2 \Rightarrow \frac{\cancel{4}^{2}\cos\theta}{\cancel{2}} = 1$

$\therefore 2\cos\theta = 1$

$\cos\theta = \frac{1}{2} \Rightarrow \theta = \cos^{-1}(\frac{1}{2})$

$\theta = \frac{\pi}{3}$ or $60°$

Substitute θ value in eq ①

$|\phi_1 - \phi_2| = \frac{\pi/3}{2}$

$\therefore |\phi_1 - \phi_2| = \frac{\pi}{6}$ or $30°$

Hence the angle of intersection $= \frac{\pi}{6}$ or $30°$

4] $r = \sin\theta + \cos\theta$ and $r = 2\sin\theta$

Soln: Taking log on b.s.

$\log r = \log(\sin\theta + \cos\theta)$

$\log r = \log(2\sin\theta)$

$\log r = \log 2 + \log\sin\theta$

Diff w.r.t. θ.

$\frac{1}{r}\frac{dr}{d\theta} = \frac{1}{\sin\theta + \cos\theta}(\cos\theta - \sin\theta)$

$\div$ by num & deno $\cos\theta$ (RHS)

$\cot\phi_1 = \frac{\frac{\cos\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta}}$

$\cot\phi_1 = \frac{1-\tan\theta}{1+\tan\theta}$

$\cot\phi_1 = \cot(\pi/4 + \theta)$

$\phi_1 = \pi/4 + \theta$

$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{\sin\theta}(\cos\theta)$

$\frac{1}{r}\frac{dr}{d\theta} = \frac{\cos\theta}{\sin\theta}$

$\cot\phi_2 = \cot\theta$

$\phi_2 = \theta$

$\therefore$ Angle of intersection $|\phi_1 - \phi_2| = |\frac{\pi}{4} + \theta - \theta|$

$\therefore |\phi_1 - \phi_2| = \frac{\pi}{4}$

Hence the angle of intersection is $\frac{\pi}{4}$

5] $r = \frac{a\theta}{1+\theta}$ and $r = \frac{a}{1+\theta^2}$

Soln :- Taking log on b.s

$\log r = \log\left(\frac{a\theta}{1+\theta}\right)$ $\qquad$ $\log r = \log\left(\frac{a}{1+\theta^2}\right)$

$\log r = \log(a\theta) - \log(1+\theta)$. $\qquad$ $\log r = \log a - \log(1+\theta^2)$

$\log r = \log a + \log\theta - \log(1+\theta)$

Diff w.r.t. θ

$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{\theta} - \frac{1}{1+\theta}$ $\qquad$ $\frac{1}{r}\frac{dr}{d\theta} = 0 - \frac{1}{(1+\theta^2)}(2\theta)$

$\cot\phi_1 = \frac{1}{\theta(1+\theta)}$ —① $\qquad$ $\cot\phi_2 = \frac{-2\theta}{1+\theta^2}$ —②

OR

$\tan\phi_1 = \theta + \theta^2$ $\qquad$ $\tan\phi_2 = \frac{1+\theta^2}{-2\theta}$

Now consider. $r = \frac{a\theta}{1+\theta}$ and $r = \frac{a}{1+\theta^2}$

$\frac{a\theta}{1+\theta} = \frac{a}{1+\theta^2}$

$a\theta(1+\theta^2) = a(1+\theta)$

$\cancel{a\theta} + a\theta^3 = a + \cancel{a\theta}$

$\frac{a\theta^3}{a} = 1 \Rightarrow \theta^3 = 1$

$\theta = 1$

Substitute θ value in eq① & eq②

$\cot\phi_1 = \frac{1}{1(1+1)}$ $\qquad$ $\cot\phi_2 = \frac{-2(1)}{1+1^2} = \frac{-2}{2}$

$\cot\phi_1 = \frac{1}{2}$ $\qquad$ $\cot\phi_2 = -1$

consider. $\cot(\phi_1 - \phi_2) = \dfrac{\cot\phi_1 \cdot \cot\phi_2 - 1}{\cot\phi_1 + \cot\phi_2}$

$$\cot(\phi_1 - \phi_2) = \frac{(1/2)(-1) - 1}{(1/2) + (-1)}$$

$$= \frac{-1/2 - 1}{1/2 - 1} = \frac{-3/2}{-1/2}$$

$$\cot(\phi_1 - \phi_2) = 3$$

$$|\phi_1 - \phi_2| = \cot^{-1}(3)$$

Pedal Equation

The eq^n of the given curves $r = f(\theta)$ expressed in terms of p and r is called as the pedal eq^n OR p-r eq^n of the curve $r = f(\theta)$.

Pedal eq^n of a polar curve is in the form $p = r\sin\phi$.

Working procedure.

1. Given Eq^n in the form $r = f(\theta)$.
2. Find the value of ϕ.
3. Then substitute ϕ in to the eq^n $p = r\sin\phi$. so that this eq^n assumes the form $p = r\,g(\theta)$.
4. Eliminate θ b/w the eq^{ns} $r = f(\theta)$ & $p = r.g(\theta)$
5. This will give us an eq^n p & r being the required pedal eq^n.

NOTE:- If we are unable to obtain ϕ explicitly in terms of θ we have to square and take the reciprocal of $p = r\sin\phi$.

$\frac{1}{p^2} = \frac{1}{r^2} \frac{1}{\sin^2\phi}$

$\frac{1}{p^2} = \frac{1}{r^2} \operatorname{cosec}^2\phi$. or $\frac{1}{p^2} = \frac{1}{r^2}(1+\cot^2\phi)$

Find the pedal Eq^{n} of the following curves

1] $\frac{2a}{r} = (1+\cos\theta)$

Soln: Taking log on b.s

$\log\left(\frac{2a}{r}\right) = \log(1+\cos\theta)$

$\log 2a - \log r = \log(1+\cos\theta)$

Diff w.r.t. θ

$0 - \frac{1}{r}\frac{dr}{d\theta} = \frac{1}{1+\cos\theta}(-\sin\theta)$

$\cancel{-}\cot\phi = \frac{\cancel{-}2\sin(\theta/2)\cos(\theta/2)}{2\cos^2(\theta/2)}$

$\cot\phi = \tan(\theta/2)$

$\cot\phi = \cot\left(\pi/2 - \theta/2\right)$

$\phi = \frac{\pi}{2} - \frac{\theta}{2}$

consider $p = r\sin\phi$. and substitute the value of ϕ we have

$p = r\sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right)$

$p = r\cos(\theta/2)$

Now, we have $\frac{2a}{r} = (1+\cos\theta)$ — ①

$p = r\cos.(\theta/2)$ — ②

We have to eliminate θ from (1) & (2).

Eq^n (1) $\Rightarrow \frac{2a}{r} = 2\cos^2(\theta/2)$

$\frac{a}{r} = \cos^2(\theta/2)$ —— (3)

Eq^n (2) $\Rightarrow \frac{p}{r} = \cos(\theta/2)$

Hence we get.

Eq^n (3) $\Rightarrow \frac{a}{r} = \frac{p^2}{r^2} \Rightarrow p^2 = ar$.

Thus $p^2 = ar$ is the required pedal eq^n.

2]. $r(1-\cos\theta) = 2a$.

Solu:. Taking log on b.s

$\log(r(1-\cos\theta)) = \log 2a$.

$\log r + \log(1-\cos\theta) = \log 2a$.

Diff w.s.t. θ.

$\frac{1}{r}\frac{dr}{d\theta} + \frac{1}{(1-\cos\theta)}(\sin\theta) = 0$.

$\cot\phi = \frac{-\sin\theta}{1-\cos\theta}$

$\cot\phi = \frac{-2\sin(\theta/2)\cos(\theta/2)}{2\sin^2(\theta/2)}$

$\cot\phi = -\cot(\theta/2)$

$\phi = -\theta/2$.

Consider. $p = r\sin\phi$

$p = r\sin(-\theta/2)$ $\quad p = -r\sin(\theta/2)$

Now we have, $r(1-\cos\theta) = 2a$ — ①

$$p = -r\sin(\theta/2) \quad — ②$$

we have to eliminate θ from ① & ②

Eq① ① $\Rightarrow$ $r(1-\cos\theta) = 2a$

$$r \cdot 2\sin^2(\theta/2) = 2a$$

$$r \cdot \sin^2(\theta/2) = a \quad — ③$$

Eq① ② $p = -r\sin(\theta/2)$

$$\sin(\theta/2) = \frac{-p}{r}$$

Hence we get

Eq① ③ $\Rightarrow$ $r \cdot \left(\frac{p^2}{r^2}\right) = a.$

$$p^2 = ar.$$

3] $r^2 = a^2\sec 2\theta$

Solu:- Taking log on b.s

$$2\log r = 2\log a + \log(\sec 2\theta)$$

Diff w.r.t. θ.

$$\frac{2}{r}\frac{dr}{d\theta} = 0 + \frac{2\cancel{\sec 2\theta}\tan 2\theta}{\cancel{\sec 2\theta}}$$

$$\cot\phi = \tan 2\theta.$$

$$\cot\phi = \cot\left(\frac{\pi}{2} - 2\theta\right)$$

$$\phi = \frac{\pi}{2} - 2\theta$$

consider $p = r\sin\phi$

$$p = r\sin\left(\frac{\pi}{2} - 2\theta\right)$$

$$p = r\cos 2\theta$$

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Now we have $r^2 = a^2 \sec 2\theta$ — (1)

$$p = r\cos 2\theta \quad \text{— (2)}$$

Eq (2) $\Rightarrow \frac{p}{r} = \cos 2\theta \Rightarrow \frac{r}{p} = \frac{1}{\cos 2\theta}$

$$\frac{r}{p} = \sec 2\theta$$

Eq (1) $\Rightarrow r^2 = a^2\left(\frac{r}{p}\right)$

$$a^2 = pr.$$

4] $r^n = a^n \cos n\theta$

Solu:- Taking log on b.s.

$$n\log r = n\log a + \log\cos n\theta.$$

Diff w.r.t. θ.

$$\frac{n}{r}\frac{dr}{d\theta} = 0 + \frac{(-n\sin n\theta)}{\cos n\theta}$$

$$\cot\phi = -\tan n\theta.$$

$$\cot\phi = \cot\left(\frac{\pi}{2} + n\theta\right)$$

$$\phi = \frac{\pi}{2} + n\theta$$

consider $p = r\sin\phi$

$$p = r\sin\left(\frac{\pi}{2} + n\theta\right)$$

$$p = r\cos n\theta$$

Now we have $r^n = a^n\cos n\theta$ — (1)

$$p = r\cos n\theta \quad \text{— (2)}$$

we have to eliminate θ from (1) & (2).

Eq (2) $\frac{p}{r} = \cos n\theta$

Eq (1) $r^n = a^n(p/r)$ OR $r^{n+1} = pa^n$

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5] $r^m = a^m(\cos m\theta + \sin m\theta)$

Solu: Taking log on b.s.

$m \log r = m \log a + \log(\cos m\theta + \sin m\theta)$

Diff w.r.t. θ.

$$\frac{m}{r}\frac{dr}{d\theta} = 0 + \frac{1}{\cos m\theta + \sin m\theta}(-m\sin m\theta + m\cos m\theta)$$

$$m\frac{1}{r}\frac{dr}{d\theta} = \frac{\cancel{m}(\cos m\theta \mp \sin m\theta)}{\cos m\theta + \sin m\theta}$$

$\div$ by numer & deno of RHS by $\cos m\theta$.

$$\cot\phi = \frac{\frac{\cos m\theta}{\cos m\theta} - \frac{\sin m\theta}{\cos m\theta}}{\frac{\cos m\theta}{\cos m\theta} + \frac{\sin m\theta}{\cos m\theta}}$$

$$\cot\phi = \frac{1 \mp \tan m\theta}{1 + \tan m\theta}$$

$$\cot\phi = \cot(\pi/4 + m\theta)$$

$$\phi = \frac{\pi}{4} + m\theta$$

consider, $p = r\sin\phi$

$$p = r\sin\left(\frac{\pi}{4} + m\theta\right)$$

$[\sin(A+B) = \sin A\cos B + \cos A\sin B]$

$p = r\left[\sin\frac{\pi}{4}\cdot\cos m\theta + \cos\frac{\pi}{4}\sin m\theta\right]$ $\quad \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$p = r\left[\frac{1}{\sqrt{2}}\cos m\theta + \frac{1}{\sqrt{2}}\sin m\theta\right]$ $\quad \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$$p = \frac{r}{\sqrt{2}}[\cos m\theta + \sin m\theta]$$

Now we have, $r^m = a^m(\cos m\theta + \sin m\theta)$ — (1)

$$p = \frac{r}{\sqrt{2}}(\cos m\theta + \sin m\theta) \text{ — (2)}$$

we have to eliminate θ from (1) & (2)

Eqn (2) $\Rightarrow$ $\cos m\theta + \sin m\theta = \frac{p\sqrt{2}}{r}$

Eqn (1) $\Rightarrow$ $r^m = a^m\left(p\sqrt{2}/r\right)$ OR $r^{m+1} = \sqrt{2}\,a^m p$

6] $\frac{l}{r} = 1 + e\cos\theta$

Soln: Taking log on b.s

$\log l - \log r = \log(1 + e\cos\theta)$

Diff w.r.t θ

$$0 - \frac{1}{r}\frac{dr}{d\theta} = \frac{1}{1+e\cos\theta}(-e\sin\theta)$$

$$-\cot\phi = -\frac{e\sin\theta}{1+e\cos\theta}$$

$$\cot\phi = \frac{e\sin\theta}{1+e\cos\theta}$$

we cannot find ϕ explicitly.

consider. $p = r\sin\phi$.

By squaring and taking the reciprocal we have,

$$\frac{1}{p^2} = \frac{1}{r^2}\frac{1}{\sin^2\phi}.$$

$$\frac{1}{p^2} = \frac{1}{r^2}\operatorname{cosec}^2\phi \quad \text{OR} \quad \frac{1}{p^2} = \frac{1}{r^2}(1 + \cot^2\phi)$$

Substituting for $\cot\phi$ itself we have

$$\frac{1}{p^2} = \frac{1}{r^2}\left[1 + \frac{e^2\sin^2\theta}{(1+e\cos\theta)^2}\right] \quad \text{--- (1)}$$

Also we have. $\frac{l}{r} = 1 + e\cos\theta$ --- (2)

we need to eliminate θ from (1) & (2)

from (2) $\frac{l}{r} - 1 = e\cos\theta$ --- (3)

Also $e^2\sin^2\theta = e^2(1 - \cos^2\theta)$

$$e^2\sin^2\theta = e^2 - e^2\cos^2\theta$$

By using ③ we have

$$e^2 \sin^2\theta = e^2 - \left[\frac{l}{r} - 1\right]^2 \quad \text{— ④}$$

Now substituting ③ and ④ in ① we have,

$$\frac{1}{p^2} = \frac{1}{r^2}\left[1 + \frac{e^2 - \left(\frac{l}{r} - 1\right)^2}{\left(l^2/r^2\right)}\right]$$

$$\frac{1}{p^2} = \frac{1}{r^2}\left[1 + \left(r^2/l^2\right) e^2 - \left(\frac{l}{r} - 1\right)^2\right]$$

$$\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{l^2}\left[e^2 - \left(\frac{l}{r} - 1\right)^2\right]$$

$$\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{l^2}\left[e^2 - \frac{l^2}{r^2} + \frac{2l}{r} - 1\right]$$

$$\frac{1}{p^2} = \cancel{\frac{1}{r^2}} + \frac{e^2}{l^2} - \cancel{\frac{1}{r^2}} + \frac{2}{lr} - \frac{1}{l^2}$$

$$\frac{1}{p^2} = \frac{e^2 - 1}{l^2} + \frac{2}{lr}$$

7]. $r = 2(1 + \cos\theta)$

Soln:- Taking log on b.s

$$\log r = \log 2 + \log(1 + \cos\theta)$$

Diff w.r.t θ.

$$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{1 + \cos\theta}(-\sin\theta)$$

$$\cot\phi = \frac{-2\sin(\theta/2)\cdot\cos(\theta/2)}{2\cos^2(\theta/2)}$$

$$\cot\phi = -\tan(\theta/2)$$

$$\cot\phi = \cot(\pi/2 + \theta/2)$$

$$\phi = \pi/2 + \theta/2$$

consider $p = r \sin\phi$

$$p = r \sin\left(\frac{\pi}{2} + \frac{\theta}{2}\right)$$

$$p = r \cos(\theta/2)$$

Now. we have $r = 2(1+\cos\theta)$ — ①

$$p = r\cos(\theta/2) \quad — ②$$

we have to eliminate θ from ① & ②.

Eqⁿ ① $\Rightarrow$ $r = 2 \cdot 2\cos^2(\theta/2)$

$$r = 4\cos^2(\theta/2)$$

Eqⁿ ② $\Rightarrow$ $\frac{p}{r} = \cos(\theta/2)$

Now. $r = 4\frac{p^2}{r^2}$

$$r^3 = 4p^2$$

8]. $r^n = a^n \operatorname{sech} n\theta$

Solⁿ :- Taking log on b.s

$$\log r^n = \log(a^n \operatorname{sech} n\theta)$$

$$n\log r = \log a^n + \log(\operatorname{sech} n\theta)$$

$$n\log r = n\log a + \log(\operatorname{sech} n\theta)$$

Diff w.r.t θ.

$$n\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{\operatorname{sech} n\theta}(-n\operatorname{sech} n\theta \cdot \tanh n\theta)$$

$$\cancel{n}\cdot\frac{1}{r}\frac{dr}{d\theta} = \frac{-\cancel{n}\,\cancel{\operatorname{sech} n\theta}\cdot\tanh n\theta}{\cancel{\operatorname{sech} n\theta}}$$

$$\cot\phi = -\tanh n\theta \quad \text{[we can not find } \phi]$$

consider. $p = r\sin\phi$

By squring and taking the reciprocal. we have,

$$\frac{1}{p^2} = \frac{1}{r^2}\frac{1}{\sin^2\phi}$$

$$\frac{1}{p^2} = \frac{1}{r^2}\operatorname{cosec}^2\phi \quad \underline{OR} \quad \frac{1}{p^2} = \frac{1}{r^2}(1+\cot^2\phi)$$

Substituting for $\cot\phi$ itself we have

$$\frac{1}{p^2} = \frac{1}{r^2}[1+\underline{\tanh^2 n\theta}] \quad \text{— (1)}$$

Also we have, $r^n = a^n \underline{\operatorname{sech} n\theta}$ — (2)

we need to eliminate θ from (1) & (2).

from (2) $\frac{r^n}{a^n} = \underline{\operatorname{sech} n\theta}$ — (3)

Also we have $1-\tanh^2 n\theta = \operatorname{sech}^2 n\theta$

$$\tanh^2 n\theta = 1-\operatorname{sech}^2 n\theta$$

$$\tanh^2 n\theta = 1-\left[\frac{r^n}{a^n}\right]^2$$

Substituting this in the RHS of eqn (1) we get

$$\frac{1}{p^2} = \frac{1}{r^2}\left[1+\left[1-\frac{r^{2n}}{a^{2n}}\right]\right]$$

$$\underline{\frac{1}{p^2} = \frac{1}{r^2}\left[2-\frac{r^{2n}}{a^{2n}}\right]}$$

9] For the equiangular spiral $r = ae^{\theta\cot\alpha}$, a and α are constants. Show that the tangent is inclined at a constant angle with the radius vector and hence find the pedal eqn of the curve.

Soln: We have $r = ae^{\theta\cot\alpha}$

Taking log on b.s

$$\log r = \log ae^{\theta\cot\alpha} \qquad \log a^n = n\log a$$

$$\log r = \log a + \log e^{\theta\cot\alpha}$$

$$\log r = \log a + \theta\cot\alpha \log e \qquad \log e = 1.$$

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$\log r = \log a + \cot\alpha \cdot \theta$ (i)

Diff w.r.t θ.

$$\frac{1}{r}\frac{dr}{d\theta} = \cot\alpha \cdot 1$$

$$\cot\phi = \cot\alpha$$

$$\phi = \alpha.$$

$\therefore \alpha$ is constant

$\therefore$ the tangent is inclined at a constant angle with the radius vector.

consider. $p = r\sin\phi$. But $\phi = \alpha$.

$$p = r\sin\alpha.$$

This being independent of θ is the pedal eqn.

Thus $p = r\sin\alpha$ is the required pedal eqn.

10] Find the length of the perpendicular from the pole to the tangent at the point $(a, \pi/2)$ on the curve $r = a(1-\cos\theta)$.

Solu :- $r = a(1-\cos\theta)$.

Taking log on b.s

$$\log r = \log(a(1-\cos\theta))$$

$$\log r = \log a + \log(1-\cos\theta)$$

Diff w.r.t θ.

$$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{1-\cos\theta}(\sin\theta)$$

$$\cot\phi = \frac{2\sin(\theta/2)\cos(\theta/2)}{2\sin^2(\theta/2)}$$

$$\cot\phi = \cot(\theta/2)$$

$$\Rightarrow \phi = \theta/2$$

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Length of the perpendicular. $p = r \sin\phi$

$$p = r \sin(\theta/2).$$

Substituting $(r, \theta) = (a, \pi/2)$

we get $p = a \sin(\pi/4)$

$$p = \frac{a}{\sqrt{2}}$$

$p = a \sin\left(\frac{\pi/2}{2}\right)$

$p = a \sin \pi/4$

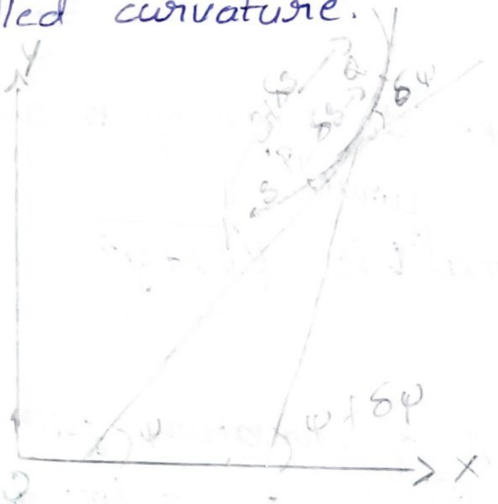
$\sin \pi/4 = \frac{1}{\sqrt{2}}$

curvature and Radius of curvature

curvature : measuring the bendness of the curve

OR

The amount of bending of a curve at given point is called curvature.



$\widehat{AP} = s$ and $\widehat{AQ} = s + \delta s$ so that $\widehat{PQ} = \delta s$.

Let ψ and $\psi + \delta\psi$ respectively ~~by~~ be the angle made by the tangent at P and Q with the initial x-axis. The angle $\delta\psi$ between the tangents is called the bending of the curve.

$$\text{curvature} : k = \frac{d\psi}{ds}$$

If $k \neq 0$ the reciprocal of the curvature is called as the radius of curvature and denoted by ρ

$$\text{Radius of curvature} = \rho = \frac{1}{k} = \frac{ds}{d\psi}$$

Expression for the radius of curvature for a cartesian curve

Given eqⁿ is in the form $y = f(x)$

$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$$

where $y_1 = \frac{dy}{dx}$ and $y_2 = \frac{d^2y}{dx^2}$

If $x = f(y)$. $\rho = \frac{(1+x_1^2)^{3/2}}{x_2}$.

where $x_1 = \frac{dx}{dy}$ and $x_2 = \frac{d^2x}{dy^2}$

NOTE :-

① We always take the sign of K and ρ to be positive.

② Length of normal curve is $y\sqrt{1+y_1^2}$

problems

① Find the radius of curvature for the curve whose intrinsic eqⁿ is $s = a \log \tan\left(\frac{\pi}{4}+\frac{\psi}{2}\right)$

Solⁿ :- $s = a \log \tan\left(\frac{\pi}{4}+\frac{\psi}{2}\right)$ and we have $\rho = \frac{ds}{d\psi}$

Differentiating w.r.t ψ.

$$\frac{ds}{d\psi} = a \cdot \frac{1}{\tan\left(\frac{\pi}{4}+\frac{\psi}{2}\right)} \sec^2\left(\frac{\pi}{4}+\frac{\psi}{2}\right) \cdot \frac{1}{2}$$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$$= \frac{a}{2} \frac{\cos\left(\frac{\pi}{4}+\frac{\psi}{2}\right)}{\sin\left(\frac{\pi}{4}+\frac{\psi}{2}\right)} \cdot \frac{1}{\cos^2\left(\frac{\pi}{4}+\frac{\psi}{2}\right)}$$

$\frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

$$= \frac{a}{2\sin\left(\frac{\pi}{4}+\frac{\psi}{2}\right) \cos\left(\frac{\pi}{4}+\frac{\psi}{2}\right)}$$

$\sec = \frac{1}{\cos\theta}$

But $2\sin\theta \cdot \cos\theta = \sin 2\theta$.

$$\therefore \frac{ds}{d\psi} = \frac{a}{\sin\left[2\left(\frac{\pi}{4}+\frac{\psi}{2}\right)\right]}.$$

$$= \frac{a}{\sin\left(\frac{\pi}{2}+\psi\right)}$$

$\sin(\pi/2 + \theta) = \cos\theta$

$\frac{1}{\cos\theta} = \sec\theta$

$$= \frac{a}{\cos\psi}.$$

$$\rho = a\sec\psi$$

2]. S.T the radius of curvature for the catenary of uniform strength $y = a\log\sec\left(\frac{x}{a}\right)$ is $a\sec(x/a)$

Soln: We have. $\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$.

Now. consider,

$$y = a\log\sec(x/a)$$

Diff w.r.t x.

$$\frac{dy}{dx} = y_1 = a \cdot \frac{1}{\sec(x/a)} \sec(x/a) \cdot \tan(x/a)(1/a)$$

$$y_1 = \tan(x/a)$$

Diff w.r.t x.

$$y_2 = \frac{1}{a}\sec^2(x/a)$$

$$\therefore \rho = \frac{[1+\tan^2(x/a)]^{3/2}}{\frac{\sec^2(x/a)}{a}}$$

$1 + \tan\theta = \sec\theta$

$$= \frac{a[\sec^2(x/a)]^{3/2}}{\sec^2(x/a)}$$

$$= \frac{a\sec^3(x/a)}{\sec^2(x/a)}$$

$$\rho = a\sec(x/a)$$

3] S.T for the catenary $y = c\cosh(x/c)$ the radius of curvature is equal to y^2/c. which is also equal to the length of the normal intercepted b/w the curve and the x-axis.

Solu :. we have $\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$

Now consider. $y = c\cosh(x/c)$

Diff w.r.t x.

$$\frac{dy}{dx} = y_1 = c.\sinh(x/c).1/c$$

$$y_1 = \sinh(x/c)$$

Again diff w.r.t x

$$y_2 = \cosh(x/c)\ 1/c$$

$\cosh^2\theta - \sinh^2\theta = 1$

$1+\sinh^2\theta = \cosh^2\theta$

$$\therefore \rho = \frac{[1+\sinh^2(x/c)]^{3/2}}{\cosh(x/c)(1/c)}$$

$$\rho = \frac{c[\cosh^2(x/c)]^{3/2}}{\cosh(x/c)}$$

$$\rho = \frac{c\cosh^3(x/c)}{\cosh(x/c)}$$

$$\rho = c\cosh^2(x/c)$$

But $\frac{y}{c} = \cosh(x/c)$ ← $y = c\cosh(x/c)$

$$\rho = c\frac{y^2}{c^2} \Rightarrow \text{So } \rho = y^2/c$$

Also we know that the length of the normal l is $y\sqrt{1+y_1^2}$

$$l = c\cosh(x/c)\sqrt{1+\sinh^2(x/c)}$$

$$l = c\cosh(x/c)\sqrt{\cosh^2(x/c)}$$

$$l = c\cosh(x/c).\cosh(x/c)$$

$$l = c\cosh^2(x/c)$$

$$l = c.\frac{y^2}{c^2}$$

$$l = y^2/c$$

4] Find the radius of curvature for the curve $y = ax^2+bx+c$ at $x = \frac{1}{2a}[\sqrt{a^2-1} - b]$.

Soln: consider. $y = ax^2+bx+c$.

Diff w.r.t x

$y_1 = 2ax+b$

Again diff w.r.t x

$y_2 = 2a$.

At the given point. $y_1 = 2a \cdot \frac{1}{2a}[\sqrt{a^2-1} - b] + b$

$y_1 = \sqrt{a^2-1}$

$y_2 = 2a$

$\therefore \rho = \frac{(1+y_1^2)^{3/2}}{y_2}$

$\rho = \frac{[1+(\sqrt{a^2-1})^2]^{3/2}}{2a} \Rightarrow \rho = \frac{(1+a^2-1)^{3/2}}{2a}$ $\frac{a^3}{2a} = \frac{a^2}{2}$

$\rho = \frac{a^2}{2}$

5]. Find the radius of curvature for the Folium of De-cartes $x^3+y^3 = 3axy$ at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ on it.

Soln:- ~~carte~~ consider. $x^3+y^3 = 3axy$

use product rule

Diff w.r.t x.

$y_1' = \frac{dy}{dx}$

$3x^2+3y^2y_1' = 3a[xy_1' + y(1)]$

$3[x^2+y^2y_1'] = 3a[xy_1'+y]$

$x^2+y^2y_1' = axy_1' + ay$

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$$y^2 y_1' - axy_1' = ay - x^2$$

$$(y^2 - ax) y_1' = ay - x^2$$

$$y_1' = \frac{ay - x^2}{y^2 - ax} \quad \text{--- (1)}$$

At $\left(\frac{3a}{2}, \frac{3a}{2}\right) \Rightarrow$ $y_1 = \dfrac{\left(\frac{3a}{2}\right)a - \left(\frac{3a}{2}\right)^2}{\left(\frac{3a}{2}\right)^2 - a\left(\frac{3a}{2}\right)}$

$$y_1 = \frac{\frac{3a^2}{2} - \frac{9a^2}{4}}{\frac{9a^2}{4} - \frac{3a^2}{2}} \quad \text{or} \quad \frac{\frac{3a^2}{2} - \frac{9a^2}{4}}{-\left(\frac{9a^2}{4} + \frac{3a^2}{2}\right)}$$

$$\boxed{y_1 = -1}$$

$$d(u/v) = \frac{v(u') - u(v')}{v^2}$$

Again diff (1) w.r.t x.

$$\frac{d^2y}{dx^2} = y_2 = \frac{(y^2 - ax)(ay_1 - 2x) - (ay - x^2)(2yy_1 - a)}{(y^2 - ax)^2}$$

At $\left(\frac{3a}{2}, \frac{3a}{2}\right)$

we have. $y_2 = \dfrac{\left[\left(\frac{3a}{2}\right)^2 - a\left(\frac{3a}{2}\right)\right]\left[ay_1 - 2\left(\frac{3a}{2}\right)\right] - \left[a\left(\frac{3a}{2}\right) - \left(\frac{3a}{2}\right)^2\right]\left[2\left(\frac{3a}{2}\right)y_1 - a\right]}{\left[\left(\frac{3a}{2}\right)^2 - a\left(\frac{3a}{2}\right)\right]^2}$

$$y_2 = \frac{\left[\frac{9a^2}{4} - \frac{3a^2}{2}\right]\left[ay_1 - 3a\right] - \left[\frac{3a^2}{2} - \frac{9a^2}{4}\right]\left[3ay_1 - a\right]}{\left[\frac{9a^2}{4} - \frac{3a^2}{2}\right]^2}$$

we have. $\dfrac{9a^2}{4} - \dfrac{3a^2}{2} = \dfrac{9a^2 - 6a^2}{4} = \dfrac{3a^2}{4}$

$$\frac{3a^2}{2} - \frac{9a^2}{4} = \frac{6a^2 - 9a^2}{4} = \frac{-3a^2}{4}$$

$$y_2 = \frac{\left(\frac{3a^2}{4}\right)[a(-1) - 3a] - \left[-\frac{3a^2}{4}\right)(3a(-1) - a)}{\left(\frac{3a^2}{4}\right)^2}$$

$$y_2 = \frac{\left(\frac{3a^2}{4}\right)[-4a) - \left(-\frac{3a^2}{4}\right)(-4a)}{\frac{9a^4}{16}}$$

$$y_2 = \frac{(-3a^3 - 3a^3)16}{9a^4} \Rightarrow y_2 = \frac{(-6a^3)16}{9a^4}$$

$$y_2 = \frac{-96a^3}{9a^4} \Rightarrow \boxed{y_2 = \frac{-32}{3a}}$$

We have, $\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$

$$\rho = \frac{(1+1)^{3/2}}{-32/3a} \Rightarrow \rho = \frac{(2)^{3/2}(3a)}{-32}$$

Fractional Exponent rule

$a^{m/n} = \sqrt[n]{a^m}$

$2^{3/2} = \sqrt[2]{2^3} = \sqrt{8}$

$$\rho = \frac{\sqrt{8} \times 3a}{-32} \Rightarrow \rho = \frac{(\sqrt{4 \times 2})\,3a}{-32}$$

$\sqrt{32}\,\sqrt{32} = \sqrt{16\times 2}\,\sqrt{16\times 2}$

$= 4\sqrt{2}\,4\sqrt{2}$

$$\rho = \frac{-2\sqrt{2} \cdot 3a}{\sqrt{32} \cdot \sqrt{32}} \Rightarrow \rho = \frac{-2\sqrt{2} \cdot 3a}{4\sqrt{2} \cdot 4\sqrt{2}}$$

$$\rho = \frac{-3a}{8\sqrt{2}}$$

6] Find the radius of curvature of the curve $x = a\log(\sec t + \tan t)$, $y = a\sec t$.

Solu: $x = a\log[\sec t + \tan t]$

Diff w.r.t. t

$$\frac{dx}{dt} = a\frac{1}{\sec t + \tan t}[\sec t \cdot \tan t + \sec^2 t]$$

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$$\frac{dx}{dt} = \frac{a \sec t\,[\tan t + \sec t]}{\sec t + \tan t}$$

$$\frac{dx}{dt} = a \sec t$$

Also $y = a \sec t$

Diff w.r.t t

$$\frac{dy}{dt} = a \sec t . \tan t$$

$$y_1 = \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$= \frac{a \sec t . \tan t}{a \sec t}$$

$$y_1 = \tan t.$$

Diff w.r.t x.

$$y_2 = \sec^2 t . \frac{dt}{dx}$$

$$y_2 = \sec^2 t . \frac{1}{a \sec t}$$

$$y_2 = \frac{\sec t}{a}$$

$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2} = \frac{(1+\tan^2 t)^{3/2}}{\frac{\sec t}{a}} = \frac{a(\sec^2 t)^{3/2}}{\sec t}$$

$$= \frac{a \sec^3 t}{\sec t}$$

$$\rho = a \sec^2 t$$

7] Find the radius of curvature of the tract-rix. $x = a[\cos t + \log \tan(t/2)]$, $y = a \sin t$

Soln:- $x = a[\cos t + \log \tan(t/2)]$

Diff w.r.t t

$$\frac{dx}{dt} = a\left[-\sin t + \frac{1}{\tan(t/2)} \sec^2(t/2) \cdot \frac{1}{2}\right]$$

$$= a\left[-\sin t + \frac{\cos(t/2)}{\sin(t/2)} \frac{1}{2\cos^2(t/2)}\right]$$

$$= a\left[-\sin t + \frac{1}{2\sin(t/2)\cos(t/2)}\right]$$ $2\sin\theta\cos\theta = \sin 2\theta$

$$= a\left[-\sin t + \frac{1}{\sin 2(t/2)}\right]$$

$$= a\left[-\sin t + \frac{1}{\sin t}\right]$$

$\sin^2\theta + \cos^2\theta = 1$

$$= a\left[\frac{-\sin^2 t + 1}{\sin t}\right]$$

$\cos^2\theta = 1 - \sin^2\theta$

$$= a \cdot \frac{\cos^2 t}{\sin t}$$

$$\frac{dx}{dt} = a \cdot \cos^2 t \cdot \operatorname{cosec} t.$$ $\frac{1}{\sin\theta} = \operatorname{cosec} x$

Also $y = a \sin t$

Diff w.r.t. t.

$$\frac{dy}{dt} = a\cos t$$

$$y_1 = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{a\cos t}{a\cos^2 t \cdot \operatorname{cosec} t}$$

$$y_1 = \frac{1}{\cos t \cdot \frac{1}{\sin t}} \Rightarrow y_1 = \frac{\sin t}{\cos t}$$

$y_1 = \tan t.$

Diff w.r.t x

$$y_2 = \sec^2 t \cdot \frac{dt}{dx}$$

$$y_2 = \sec^2 t \frac{1}{a\cos^2 t \cdot \operatorname{cosec} t}$$

$$y_2 = \frac{\sec^2 t}{a \frac{1}{\sec^2 t} \cdot \frac{1}{\sin t}}$$

$$y_2 = \frac{\sec^4 t \cdot \sin t}{a}$$

$\frac{1}{\cos\theta} = \sec\theta$

$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2} = \frac{(1+\tan^2 t)^{3/2}}{\frac{\sec^4 t \cdot \sin t}{a}}$$

$$\rho = \frac{a(\sec^2 t)^{3/2}}{\sec^4 t \cdot \sin t} \Rightarrow \rho = \frac{a}{\sec t \cdot \sin t}$$

$$\rho = \frac{a}{\frac{1}{\cos t} \cdot \sin t} \Rightarrow \rho = \frac{a \cos t}{\sin t}$$

$$\rho = a \cot t$$

8] Find the radius of curvature of the astroid $x = a\cos^3\theta$, $y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$.

Soln: $x = a\cos^3\theta$ $\qquad$ $y = a\sin^3\theta$

Diff w.r.t θ $\qquad$ Diff w.r.t θ

$$\frac{dx}{d\theta} = -3a\cos^2\theta \sin\theta \qquad \frac{dy}{d\theta} = 3a\sin^2\theta \cos\theta$$

$$y_1 = \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{3a\sin^2\theta \cdot \cos\theta}{-3a\cos^2\theta \cdot \sin\theta} = -\frac{\sin\theta}{\cos\theta}$$

$$y_1 = -\tan\theta$$

Diff w.r.t. x.

$$y_2 = -\sec^2\theta \cdot \frac{d\theta}{dx} \Rightarrow y_2 = -\sec^2\theta \frac{1}{-3a\cos^2\theta \cdot \sin\theta}$$

$$y_2 = \frac{\sec^2\theta \cdot \sec^2\theta \cdot \operatorname{cosec}\theta}{3a}$$

$$y_2 = \frac{\sec^4\theta \cdot \operatorname{cosec}\theta}{3a}$$

$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2} = \frac{(1+\tan^2\theta)^{3/2}}{\frac{\sec^4\theta \cdot \operatorname{cosec}\theta}{3a}}$$

$$\rho = \frac{3a \cdot (\sec^2\theta)^{3/2}}{\sec^4\theta \cdot \operatorname{cosec}\theta} \Rightarrow \rho = \frac{3a}{\sec\theta \cdot \operatorname{cosec}\theta}$$

$$\rho = 3a\cos\theta \cdot \sin\theta$$

at $\theta = \frac{\pi}{4}$. $\rho = 3a\cos\frac{\pi}{4} \cdot \sin\frac{\pi}{4}$

$$\rho = 3a \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}}$$

$$\rho = \frac{3a}{2}$$

9] S.T the radius of curvature of the curve $x = a(\cos t + t\sin t)$, $y = a(\sin t - t\cos t)$ is "at"

Soln: $x = a(\cos t + t\sin t)$ product rule

Diff w.r.t. t

$$\frac{dx}{dt} = a[-\sin t + t\cos t + \sin t(1)]$$

$$\frac{dx}{dx} = at\cos t$$

$y = a(\sin t - t\cos t)$ product rule

Diff w.r.t. t.

$$\frac{dy}{dt} = a[\cos t - t(-\sin t) - \cos t(1)]$$

$$\frac{dy}{dt} = at\sin t$$

$$y_1 = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{at\sin t}{at\cos t}$$

$$y_1 = \tan t$$

Diff w.r.t. x

$y_2 = \sec^2 t \cdot \frac{dt}{dx} \Rightarrow y_2 = \sec^2 t \frac{1}{at\cos t} = \frac{\sec^2 t \cos}{at}$

$y_2 = \frac{\sec^3 t}{at}$ $\quad \frac{1}{\cos t} = \sec t.$

$\rho = \frac{(1+\cancel{\tan^2} y_1^2)^{3/2}}{y_2} = \frac{(1+\tan^2 t)^{3/2}}{\frac{\sec^3 t}{at}}$

$\rho = \frac{at (\sec^2 t)^{3/2}}{\sec^3 t} \Rightarrow \rho = at$

10]. S.T the radius of curvature at any point θ on the cycloid $x = a(\theta + \sin\theta)$. $y = a(1-\cos\theta)$. is $4a \cos \theta/2$.

Soln:- $x = a(\theta + \sin\theta)$ $\qquad y = a(1-\cos\theta)$

Diff w.s.t θ $\qquad$ Diff. w.s.t θ.

$\frac{dx}{d\theta} = a[1+\cos\theta]$ $\qquad \frac{dy}{d\theta} = a(0 + \sin\theta)$

$\frac{dy}{d\theta} = a\sin\theta$

$y_1 = \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a \sin\theta}{a[1+\cos\theta]}$

$y_1 = \frac{2\sin(\theta/2)\cos(\theta/2)}{2\cos^2(\theta/2)} = \frac{\sin(\theta/2)}{\cos(\theta/2)}$

$y_1 = \tan(\theta/2)$

Diff w.s.t. x $\quad y_2 = \sec^2 \theta/2 \cdot 1/2 \cdot \frac{d\theta}{dx}$

$y_2 = \sec^2(\theta/2)\, 1/2 \cdot \frac{1}{a(1+\cos\theta)}$

$y_2 = \frac{\sec^2(\theta/2)}{2a\, 2\cos^2(\theta/2)}$

$$y_2 = \frac{\sec^2(\theta/2)}{4a\cos^2(\theta/2)} \qquad \frac{1}{\cos^2(\theta/2)} = \sec^2(\theta/2).$$

$$y_2 = \frac{\sec^4(\theta/2)}{4a}$$

$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2} = \frac{[1+\tan^2(\theta/2)]^{3/2}}{\frac{\sec^4(\theta/2)}{4a}}$$

$$\rho = \frac{4a\,[\sec^2(\theta/2)]^{3/2}}{\sec^4(\theta/2)}$$

$$\rho = \frac{4a}{\sec(\theta/2)} \Rightarrow \rho = 4a\cos(\theta/2)$$

11] Find the radius of curvature for the curve $y^2 = \frac{4a^2(2a-x)}{x}$ where the curve meets the x-axis

Soln :- If the curve meets the x-axis then $y=0$.

$$\therefore \frac{4a^2(2a-x)}{x} = 0.$$

$$4a^2(2a-x) = 0$$

$$8a^3 - 4xa^2 = 0 \Rightarrow a^2(8a - 4x) = 0$$

$$8a - 4x = \frac{0}{a^2} \Rightarrow -4x = -8a$$

$$2a - x = 0/4a^2. \qquad 2a = x$$

$$\therefore x = 2a$$

Thus $(2a, 0)$ is the point on the curve at which we have to find ρ.

The given equation can be put in the form

$$y^2 = \frac{4a^2(2a-x)}{x}$$

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$$y^2 = \frac{8a^3 - 4a^2x}{x} \Rightarrow y^2 = \frac{8a^3}{x} - \frac{4a^2x}{x}$$

$$y^2 = \frac{8a^3}{x} - 4a^2$$

Diff w.r.t. x we have. $\left[\frac{x(0) - 8a^3(1)}{x^2}\right]$

$$2yy_1 = 8a^3\left(\frac{-1}{x^2}\right) - 0 \qquad -\frac{8a^3}{x^2}$$

$$y_1 = -\frac{\cancel{8}^{4}a^3}{x^2} \times \frac{1}{\cancel{2}y}$$

$$y_1 = \frac{-4a^3}{x^2y}$$

At $(2a, 0)$. $y_1 = \frac{-4a^3}{(2a)^2(0)} = \infty$

y_1 becomes infinity and hence we have to consider dx/dy

Let $x_1 = \frac{dx}{dy} = -\frac{x^2y}{4a^3}$ — ①

At $(2a, 0)$. $x_1 = \frac{-(2a)^2(0)}{4a^3} = 0$

$$x_1 = 0$$

Aggain diff ① w.r.t y

$$x_2 = \frac{d^2x}{dy^2} = \frac{-1}{4a^3}\left[x^2(1) + y\, 2x\, x_1\right]$$

$$x_2 = \frac{-1}{4a^3}\left[x^2 + 2xyx_1\right]$$

At $(2a, 0)$. $x_2 = -\frac{1}{4a^3}\left[(2a)^2 + 2(2a)(0)(0)\right]$

$$x_2 = \frac{-4a^2}{4a^3}$$

$$x_2 = -1/a$$

we have $\rho = \frac{(1+x_1^2)^{3/2}}{x_2}$

$$\rho = \frac{(1+(0)^2)^{3/2}}{-1/a} \Rightarrow \rho = \frac{1 \times a}{-1}$$

$$\rho = -a$$

12] Find the radius of curvature for the curve $x^2 y = a(x^2+y^2)$ at the point $(-2a, 2a)$

Soln:- Consider $x^2 y = a(x^2+y^2)$

Diff. w.r.t. x.

$$x^2 y_1 + y \cdot 2x = a[2x + 2yy_1]$$

$$x^2 y_1 + 2yx = 2ax + 2ayy_1$$

$$(x^2 y_1 - 2ayy_1) = 2ax - 2yx$$

$$y_1(x^2 - 2ay) = 2ax - 2yx$$

$$y_1 = \frac{2ax - 2yx}{x^2 - 2ay}$$

At $(-2a, 2a)$. $y_1 = \frac{2a(-2a) - 2(2a)(-2a)}{(-2a)^2 - 2a(2a)}$

$$y_1 = \frac{-4a^2 + 8a^2}{4a^2 - 4a^2} = \infty$$

$$y_1 = \infty$$

y_1 becomes infinity and hence we have to consider -s dx/dy.

Let. $x_1 = \frac{dx}{dy} = \frac{x^2 - 2ay}{2ax - 2xy}$ — ①

At $(-2a, 2a)$, $x_1 = 0$

Again diff ① w.r.t y.

$$x_2 = \frac{d^2x}{dy^2} = \frac{(2ax - 2xy)(2xx_1 - 2a) - (x^2 - 2ay)(2ax_1 - 2x - 2x_1y)}{(2ax - 2xy)^2}$$

At $(-2a, 2a)$.. $(2ax - 2xy) = (2a(-2a) - 2(-2a)(2a))$

$= -4a^2 + 8a^2$

$(2ax - 2xy) = 4a^2$

$(x^2 - 2ay) = 0$

$x_2 = \frac{(4a^2)(-2a)}{16a^4}$ $\Rightarrow$ $x_2 = \frac{-8a^3}{16_2a^4}$

$x_2 = \frac{-1}{2a}$

We have. $\rho = \frac{(1 + x_1^2)^{3/2}}{x_2}$

$\rho = \frac{(1 + 0^2)^{3/2}}{-1/2a}$

$\rho = \frac{1 \times 2a}{-1}$

$\rho = -2a$

* Extreme values of a single variable cost and revenue

Rate of changes in Economic Marginals

Marginal profit, Marginal revenue and marginal cost are the rates of change of profit, revenue and cost with respect to time.

The following table shows the various function and their meaning.

Cost function	C or $C(x)$	Marginal cost	$\frac{dC}{dx}$
Revenue function	R or $R(x)$	Marginal Revenue	$\frac{dR}{dx}$
Profit function	$P = R - C$ or $P(x) = R(x) - C(x)$	Marginal profit	$\frac{dP}{dx} = \frac{dR}{dx} - \frac{dC}{dx}$

* Problems -

1] A company can manufacture x items at a cost of $c(x)$ dollars, a sales revenue of $r(x)$ dollars and a profit of $p(x) = r(x) - c(x)$ dollars (everything in thousands). Find $\frac{dc}{dt}$, $\frac{dr}{dt}$ and $\frac{dp}{dt}$ for the following of x and $\frac{dx}{dt}$.

a] $r(x) = 9x$, $c(x) = x^3 - 6x^2 + 15x$ and $\frac{dx}{dt} = 0.1$ when $x = 2$

b] $r(x) = 70x$, $c(x) = x^3 - 6x^2 + \frac{45}{x}$ and $\frac{dx}{dt} = 0.05$ when $x = 1.5$

a] $r(x) = 9x$

$$\frac{dr}{dt} = 9 \cdot \frac{dx}{dt}$$

given $\frac{dx}{dt} = 0.1$ when $x = 2$

$$\Rightarrow \frac{dr}{dt} = 9(0.1) = 0.9$$

$$\boxed{\frac{dr}{dt} = 0.9}$$

$c(x) = x^3 - 6x^2 + 15x$

$$\frac{dc}{dt} = 3x^2 \frac{dx}{dt} - 12x \cdot \frac{dx}{dt} + 15 \frac{dx}{dt}$$

$$\frac{dc}{dt} = 3(4)(0.1) - 12(2)(0.1) + 15(0.1) \quad \left[\frac{dx}{dt} = 0.1 \text{ when } x = 2\right]$$

$$\boxed{\frac{dc}{dt} = 0.3}$$

$$\frac{dP}{dt} = \frac{dr}{dt} - \frac{dc}{dt}$$

$$= 0.9 - 0.3$$

$$\boxed{\frac{dP}{dt} = 0.6}$$

b] given $r(x) = 70x$

$$\frac{dr}{dt} = 70 \frac{dx}{dt}$$

$$\frac{dr}{dt} = 70(0.05) \quad \left[\frac{dx}{dt} = 0.05 \text{ when } x = 1.5\right]$$

$$\boxed{\frac{dr}{dt} = 3.5}$$

given $c(x) = x^3 - 6x^2 + \frac{45}{x}$

$$\frac{dc}{dt} = 3x^2 \cdot \frac{dx}{dt} - 12x \cdot \frac{dx}{dt} - \frac{1}{x^2}(45) \frac{dx}{dt}$$

$$\frac{dc}{dt} = 3(1.5)^2(0.05) - 12(1.5) 0.05 - \frac{45(0.05)}{(1.5)^2}$$

$$\frac{dc}{dt} = 0.3375 - 0.9 - 1$$

$$\boxed{\frac{dc}{dt} = -1.5625}$$

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$$\frac{dP}{dt} = \frac{dR}{dt} - \frac{dC}{dt}$$

$$= 3.5 - (-1.5625)$$

$$\boxed{\frac{dP}{dt} = 5.0625}$$

Marginal cost

2] Suppose it costs $c(x) = x^3 - 6x^2 + 15x$ dollars to produce x radiators when 8 to 30 radiators are produced. Your shop currently produces 10 radiators a day. About how much extra will it cost to produce one more radiator a day?

Given $c(x) = x^3 - 6x^2 + 15x \rightarrow (1)$

The marginal cost when x radiators are produce is $c'(x)$

from eq (1), $c'(x) = 3x^2 - 12x + 15$

The cost of producing one more radiation a day when 10 are produced is about $c'(10)$

$$\Rightarrow c'(10) = 3(10)^2 - 12(10) + 15$$

$$\boxed{c'(10) = 195}$$

$\therefore$ The additional cost will be about \$195.

Marginal revenue

3] Given that the marginal revenue $r(x) = x^3 - 3x^2 + 12x$ from selling x thousand candy bars with $5 \le x \le 20$. If you are currently selling 10 thousand candy bars a week, about how much you can repeat your revenue to increase the sales to 11 thousand bars a week.

Given $r(x) = x^3 - 3x^2 + 12x \rightarrow (1)$

The marginal revenue when x thousand are sold is

from eq (1), $r'(x) = 3x^2 - 6x + 12 \rightarrow (2)$

As with marginal cost, the marginal revenue function estimates the increase in revenue that will result from selling one additional unit.

Also, if you are currently selling 10 thousand candy bars a week, then you can expect your revenue to increase by about, $R'(10)$

$$R'(10) = 3(10)^2 - 6(10) + 12$$

$$R'(10) = 252$$

$\boxed{R'(10) = \$252}$, if you increase sales to 11 thousand bars a week.