Application of root finding - ion concentration

while trying to find the acidity of a saturated solution of magnesium hydroxide in hydrochloric acid, we derive the equation

$$\frac{3.64 \times 10^{-11}}{[H_3O^+]^2} = [H_3O^+] + 3.6 \times 10^{-4} \quad \text{--- (1)}$$

for the hydronium ion concentration $[H_3O^+]$.

To find the value of $[H_3O^+]$, set $x = 10^4 [H_3O^+]$

$$\Rightarrow [H_3O^+] = \frac{x}{10^4} = x\, 10^{-4}$$

$\therefore$ (1) becomes

$$\frac{3.64 \times 10^{-11}}{x^2 \times 10^{-8}} = x \times 10^{-4} + 3.6 \times 10^{-4}$$

xing by x^2 ~~$\times 10^{-8}$~~

$$3.64 \times 10^{-3} = x^3\, 10^{-4} + 3.6\, x^2 \times 10^{-4}$$

xing b/s by 10^4

$$3.64 \times 10 = x^3 + 3.6\, x^2$$

$$\Rightarrow x^3 + 3.6\, x^2 - 36.4 = 0$$

Let $f(x) = x^3 + 3.6x^2 - 36.4$

$f(2) = -14 < 0$

$f(3) = 23 > 0$

$\therefore$ Root lies b/w $(2, 3)$

Let $x_0 = 2.5$

Using Newton Raphson method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2.5 - \frac{f(2.5)}{f'(2.5)}$$

$f(x) = x^3 + 3.6x^2 - 36.4$

$f'(x) = 3x^2 + 7.2x$

$x_1 = 2.5 - \frac{1.725}{36.75} = 2.45306$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$x_2 = 2.45306 - \frac{f(2.45306)}{f'(2.45306)} = \cancel{2.81167}$ 2.45237

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.45237$

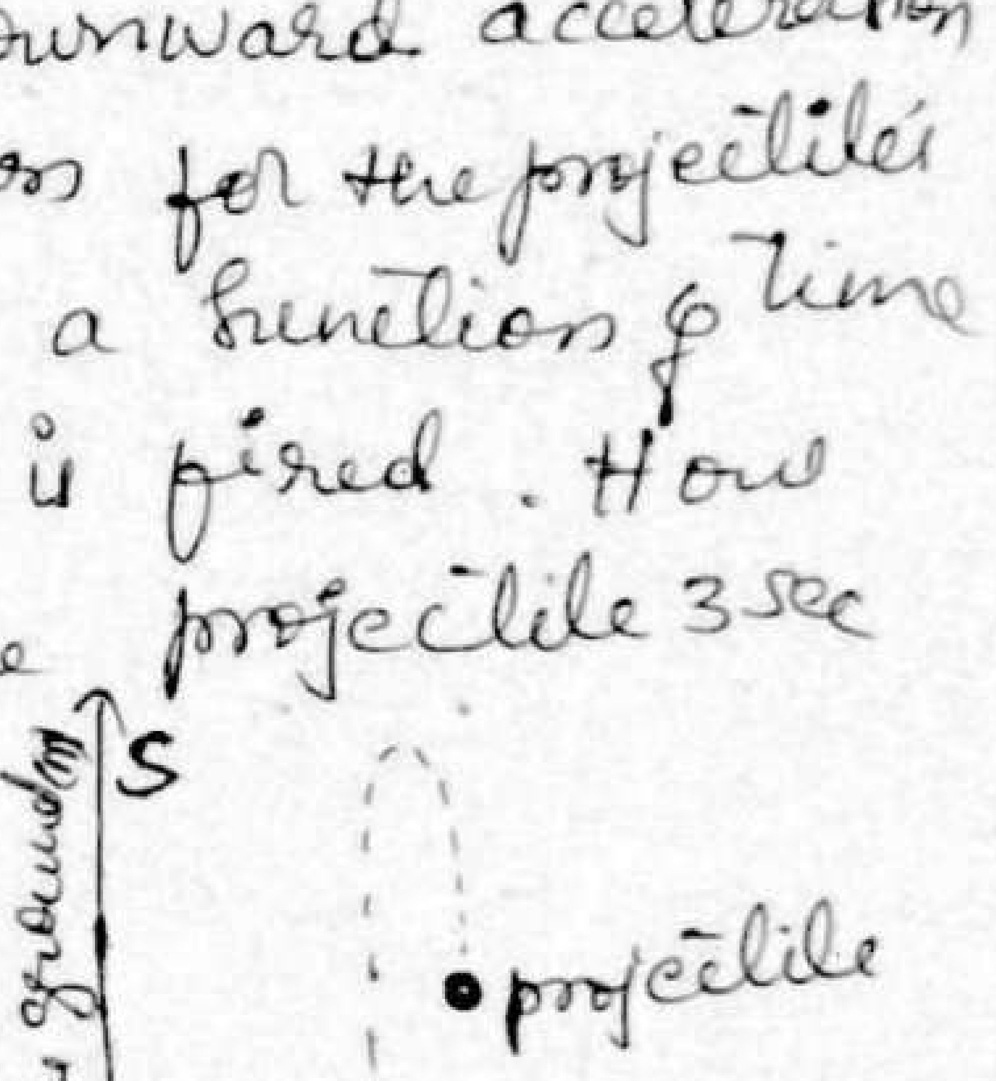
$x_2 \cong x_3$

the root is $2.4524 \cong 2.45$

$[H_3O^+] \cong 2 \times 10^{-4} = 2.45 \times 10^{-4}$

Finding a projectile's height from its acceleration, initial velocity & initial position

A heavy projectile is fired straight up from a platform 3m above the ground, with an initial velocity of 160 m/sec. Assume that the only force affecting the projectile during its flight is from gravity, which produces a downward acceleration of 9.8 m/sec^2. Find an equation for the projectile's height above the ground as a function of time t if $t = 0$ when the projectile is fired. How high above the ground is the projectile 3 sec after firing?



Sol^n; Draw figure, let s denote the projectile height above the ground (m) at time t. We assume s to be a twice differentiable function of t and represent the projectile's velocity & acceleration with the derivatives

$$v = \frac{ds}{dt} \quad \& \quad a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

since gravity acts in the direction of decreasing s in our model, the initial value problem to solve is:

the differential equ^n: $\frac{d^2s}{dt^2} = -9.8$

the initial conditions: $\frac{ds}{dt}(0) = 160$ & $s(0) = 3$.

we integrate the D.E w.r.t t to find $\frac{ds}{dt}$

$$\int \frac{d^2s}{dt^2}\,dt = \int (-9.8)\,dt$$

$$\frac{ds}{dt} = -9.8t + c_1 \quad \text{—— (1)}$$

Apply initial condition to find c_1: put $\frac{ds}{dt} = 160$ & $t = 0$ in eq (1)

$$160 = -9.8(0) + c_1$$

$\Rightarrow c_1 = 160$, using in (1)

$$\frac{ds}{dt} = -9.8t + 160 \quad \text{—— (2)}$$

Integrate Eqn (2) w.r.t t

$$S = -9.8\frac{t^2}{2} + 160t + c_2$$

$$S = -4.9t^2 + 160t + c_2 \quad \text{—— (3)}$$

Apply second initial condition to find c_2: i.e, put $S = 3$ & $t = 0$ in eq (3)

$3 = c_2 \Rightarrow c_2 = 3$, using in (3)

$$S = -4.9t^2 + 160t + 3 \quad \text{—— (4)}$$

To find the projectile height 3sec into the flight put $t = 3$ in Eqn (4)

$\therefore S = 438.9m$

Counting Principle

Sum rule & Product rule

1. Suppose a ~~hostel~~ library has 12 books on Mathematics, 10 books on Physics, 16 books on computer Science & 11 books on Electronics.

~~Suppose a~~

The Sum Rule

Suppose two tasks T_1 & T_2 are to be performed. If the task T_1 can be performed m different ways & task T_2 can be performed in n different diff ways and if these two tasks cannot be performed simultaneously, the no of ways of performing tasks is m+n ways.

Ex: Suppose a Hostel library has 12 books on Mathematics, 10 books on Physics, 16 books on computer science & 11 books on Electronics. Suppose a student wishes to choose one of these books for study. The no of ways of choosing two books is $12+10+16+11 = 49$.

Product rule

If two Tasks T_1 & T_2 are to be performed one after the other. If T_1 is performed in m different ways & T_2 can be performed in n different ways then both the tasks can be performed in $m \times n$ ways.

Ex: Suppose person has 8 shirts & 5 Ties. then he has $8 \times 5 = 40$ different ways of choosing a shirt & tie.

Permutation & combination

Permutation:

the r objects can be arranged from n distinct object refered to as permutation. It is denoted & defined as

$$P(n, r) = \frac{n!}{(n-r)!}$$

Ex: How many different strings of length 4 can be formed using the letters of the word FLOWER?

Soln: Given ~~letter~~ word contains 6 letters, all of which are distinct. We need to form a string of length 4. Arrangement of 4 letters out of 6 is given by $P(6, 4) = \frac{6!}{(6-4)!} = 360$

Combination:

Selection of r objects out of n distinct objects refered to as combination. It is denoted & defined as

$$C(n, r) = \frac{n!}{(n-r)!\, r!}$$

Ex: A women has 11 close relatives & she wishes to invite 5 of them to dinner. In how many ways can she invite them if there is no ~~st~~ restriction on the choices.

Sol: Since there is no restriction on choice, five out of 11 can be invited in

$$C(11, 5) = \frac{11!}{6!\,5!} = 462 \text{ ways.}$$

~~Baye's theorem~~

Review of probability

Random exp. ~~is~~ event the experiment when perform repeatedly giving different outcomes are called as random experiment

Ex: (i) Tossing a coin (ii) throwing a die

Sample space: the set of possible outcomes in a random experiment is called a sample space which is denoted by 'S'

Eg: (i) If coin is tossed twice
$S = \{HH, HT, TH, TT\}$
(ii) If die is thrown once
$S = \{1, 2, 3, 4, 5, 6\}$

Event : Any subset 'E' of a sample space 'S' is called an event.

Definition : If S is the sample space and E is the event of favorable case, probability of any event is given by ratio of no of favorable cases & no of possible cases.

$$P(E) = \frac{\text{No. of favorable cases}}{\text{No of possible cases.}}$$

Addition rule : If A & B are any two events $(A \cap B \neq \phi)$ of S which not mutually exclusive then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Conditional Probability : If A & B be the two events. Probability of happening of the event B when the event A has already happened is called conditional probability denoted by $P(B|A)$.

ie, $P(B|A)$ is probability of B for given A

Multiplication rule :

$$P(A \cap B) = P(A) \cdot P(B|A) \text{ where } P(A) > 0$$

Bayes theorem

Let $A_1, A_2, A_3, \ldots, A_n$ be a set of exhaustive & mutually exclusive events of the sample space S with $P(A_i) \neq 0$ for each i. If $A \subset \bigcup_{i=1}^{n} A_i$ with $P(A) \neq 0$ then

$$P(A_i|A) = \frac{P(A_i)\, P(A|A_i)}{\sum_{i=1}^{n} P(A_i)\, P(A|A_i)}$$

① A chance that a doctor will diagnose a disease correctly is 60%. The chance that a patient will die after correct diagnosis is 40% & the chance of death by wrong diagnosis is 70%. If a patient dies, what is the chance that his disease was correctly diagnosed!

Soln: Let A be event of correct diagnosis & B be the event of wrong diagnosis by doctor. $\therefore P(A) = 0.6$ & $P(B) = 0.4$

Let E be the event that patient dies.

$\therefore P(E|A) = 0.4$ & $P(E|B) = 0.7$

we have to find $P(A|E)$ & by Bayes theorem,

$$P(A|E) = \frac{P(A)\, P(E|A)}{P(A)\, P(E|A) + P(B) \cdot P(E|B)} = \frac{(0.6)(0.4)}{(0.6)(0.4) + (0.4)(0.7)}$$

thus $P(A|E) = 0.416$. chances of patient dies by still disease was correctly diagnosed is 41.6%.